

THE ELLSBERG PARADOX AND THE WEIGHT OF ARGUMENTS

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Standard Approach: Maximize Expected Utility

- Expected utility of an action: the sum of the products of multiplying (1) the probability of each circumstance given an action by (2) the utility for that action
- Maximize expected utility: act so that expected utility is as great as possible.
- If expected utilities of actions are equal, then you should be indifferent.

THE ELLSBERG PARADOX

- . Paradox for MEU.
- . There is a box with-
 - 1/3 black balls
 - Between 0 and 2/3 green balls
 - Between 0 and 2/3 red balls
- . There are two choices between bets on a randomly selected ball from the box.

	BLACK	GREEN	RED
A	£100	£0	£0
B	£0	£100	£0

A: “The ball will be black.” B: “The ball will be green.”

In experiments, most people prefer A to B

	BLACK	GREEN	RED
C	£100	£0	£100
D	£0	£100	£100

C: “The ball will be not be green.” D: “The ball will not be black.”

In experiments, most people prefer D to C

THE PARADOX

- The EU of betting A is greater than the EU of B iff the EU of C is greater than the EU of D.
- Why $A > B$?
 - Only one possible reason in MEU theory: more likely that the ball will be red rather than green.
- But then why not $C > D$?
- MEU: combination is irrational

BET A



BET B



BET C



BET D



PROBLEM

Nothing formally wrong or intuitively irrational.

Expected utilities CAN be equal.

Conservative solution?

EVIDENTIAL PROBABILITY

- . Developed by Henry E. Kyburg (1928-2007)
- . Provides a system whereby all probabilities are derived from information about relative frequencies.
- . Single probability for given evidence.
- . Evidential probabilities can be imprecise.
 - When information is imprecise.

SPECULATION AND DECISION

- . How do we get a decision-theory with Evidential Probabilities?
- . Speculate relative frequency information that is consistent with the Evidential Probabilities.
- . Bet *as if* we knew the relative frequencies.

EXAMPLE

- Tossing Gömböc: very imprecise prob.
 - Maybe $[0, 1]$
- Tossing a 1 euro coin: relatively precise prob.
 - Like $[0.49, 0.51]$
- Many would speculate: 0.5 ($1/2$)

SPECULATION AND DECISION

There is a pre-theoretical distinction between-

(1) Making decisions based on evidence.

(2) Making decisions based on speculation.

A difference of degrees – measure with Evidential Probabilities.

A tie-breaker *if* expected utilities are equal.

IMPRECISION AS A DECISION TOOL

- . Bet with even odds.
- . Gömböc or coin?
- . Coin, because less speculation.

THE ELLSBERG PARADOX

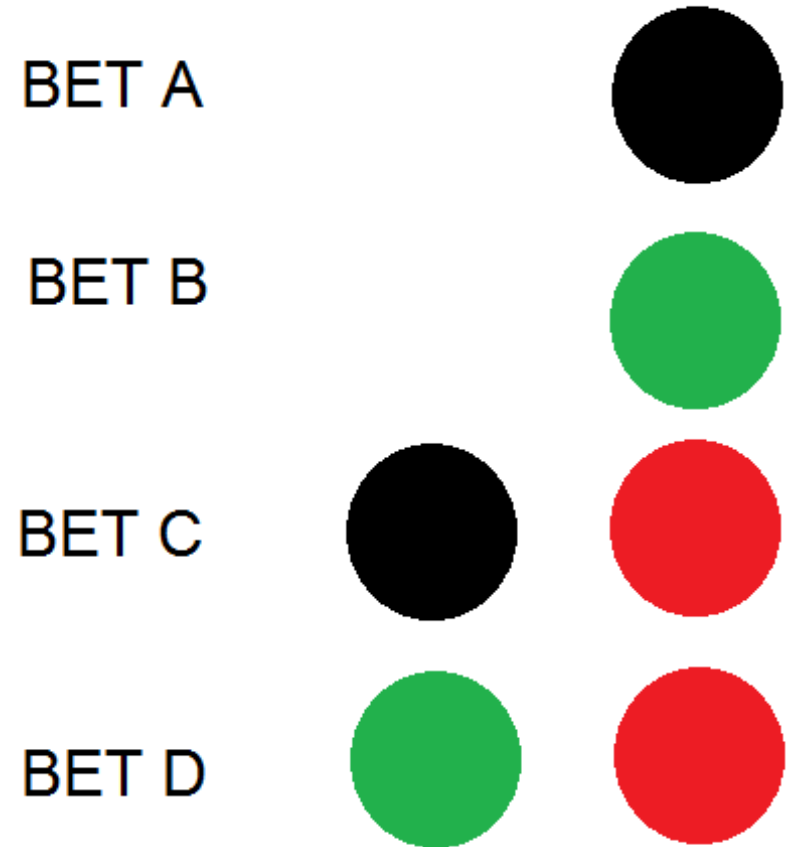
You know that $1/3$ balls are black and that $[0, 2/3]$ are green.

You might speculate that $1/3$ are green.

EU for each choice is equal.

A is less speculative than B.

D is less speculative than C.



WEIGHT OF ARGUMENTS

- John Maynard Keynes: quantity of relevant evidence (in an argument for some action) matters.
- But how?
- It can help us choose when expected utilities are equal.

CONCLUSIONS

A conservative response to the Ellsberg Paradox?

- Yes.

Is Evidential Probability AND precise decision theory?

- Yes.

Does the Weight of Argument matter? Sometimes.