#### A Discussion Game for Grounded Semantics

#### Martin Caminada

Cardiff University School of Computer Science

AF = (Ar, att)

#### Lab: Ar $\rightarrow$ {in, out, undec}

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if undec then not all attackers out and no attacker in

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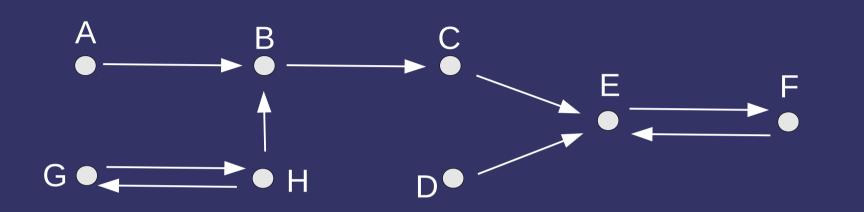
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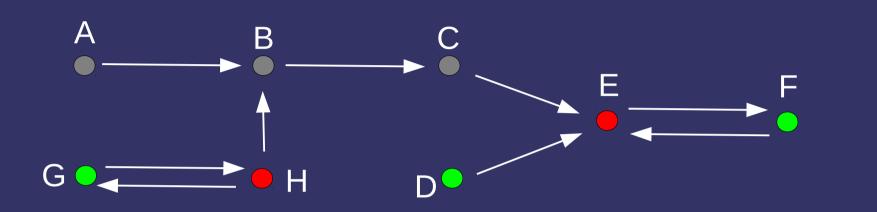
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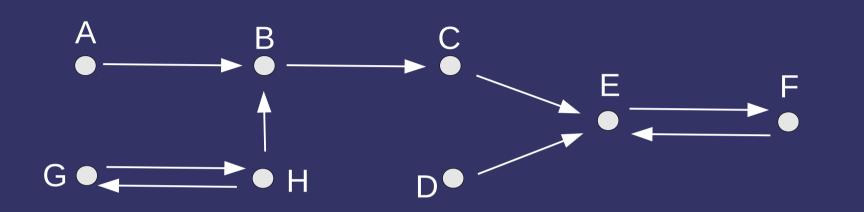
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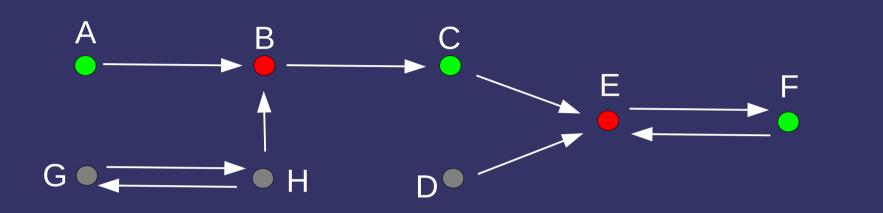
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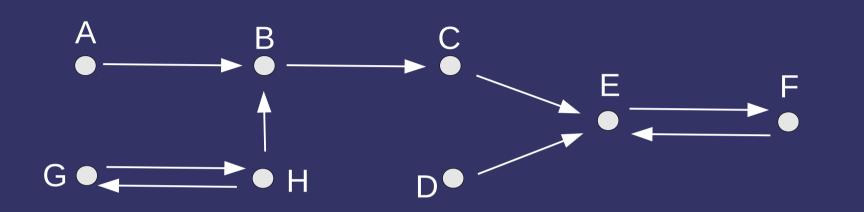
Complete labellings form a semi-lattice (w.r.t.  $\sqsubseteq$ ) with grounded at the bottom and preferreds at the top

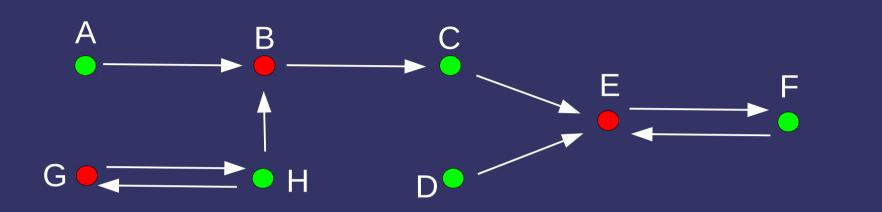


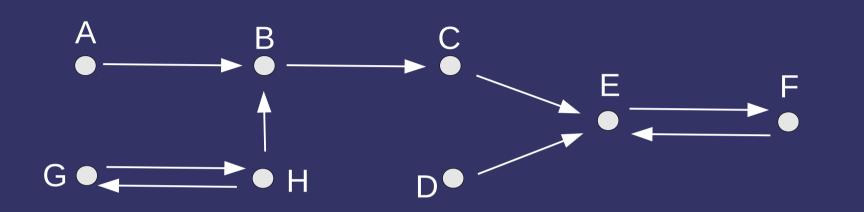


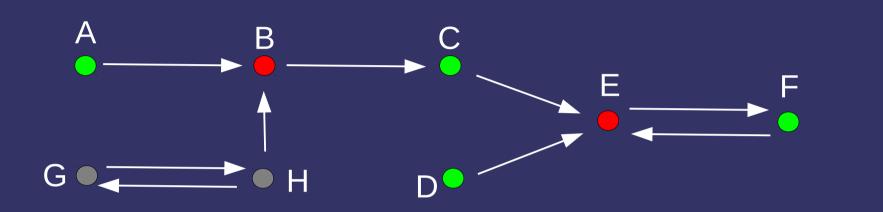














MinMax numbering *MM*: in(Lab)  $\cup$  out(Lab)  $\rightarrow$  IN $\cup$ { $\infty$ }

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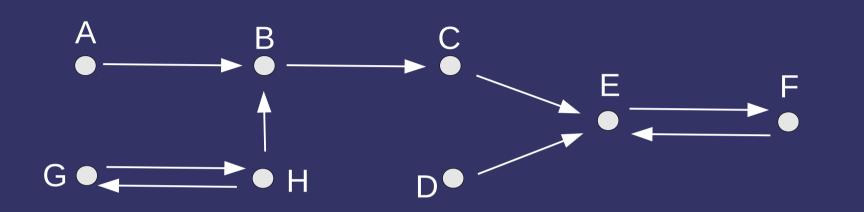
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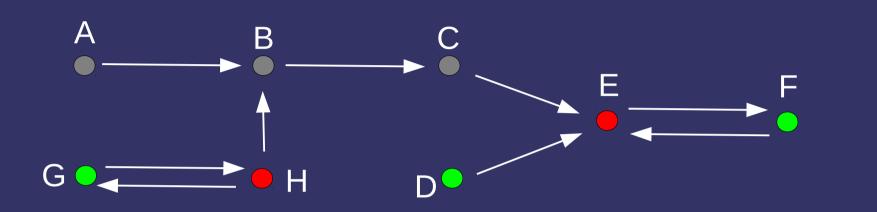
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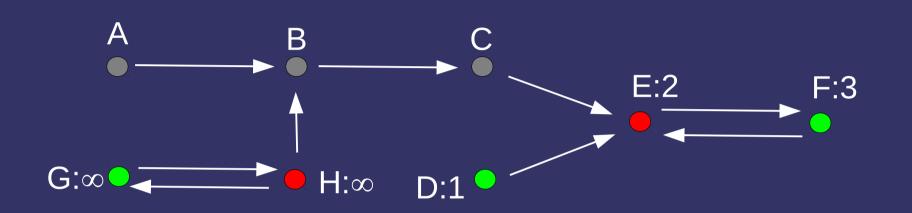
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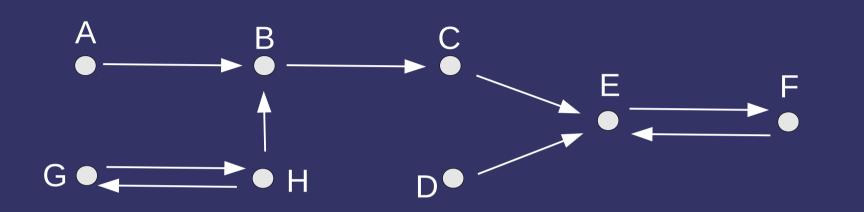
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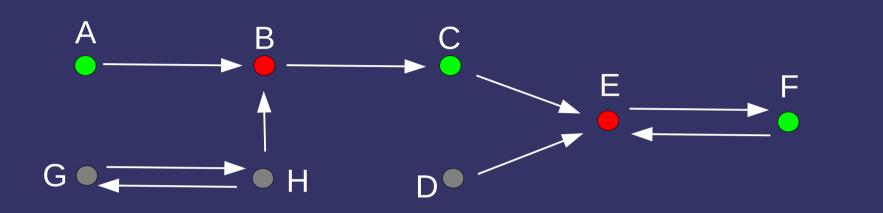
The strongly admissible labellings form a lattice with the all-undec at the bottom and the grounded at the top A in in str. adm.  $\equiv$  A in in grounded (details:COMMA14)

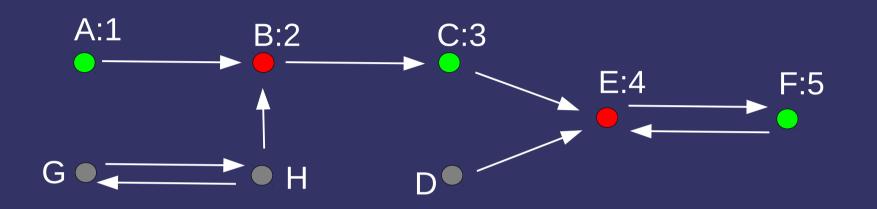


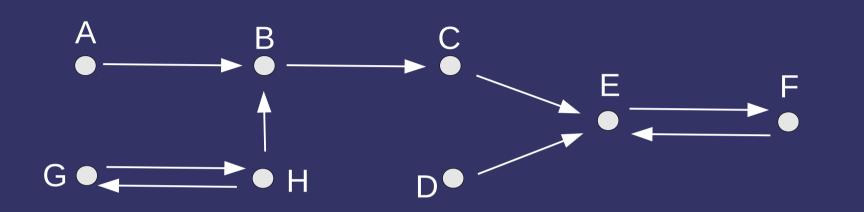


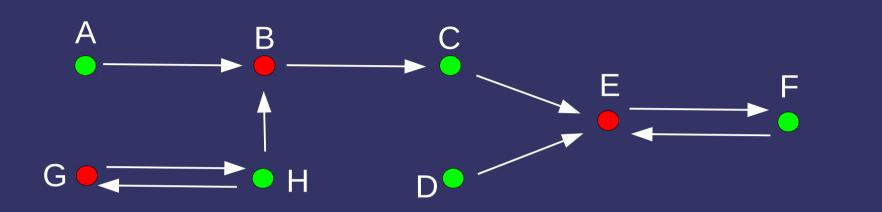


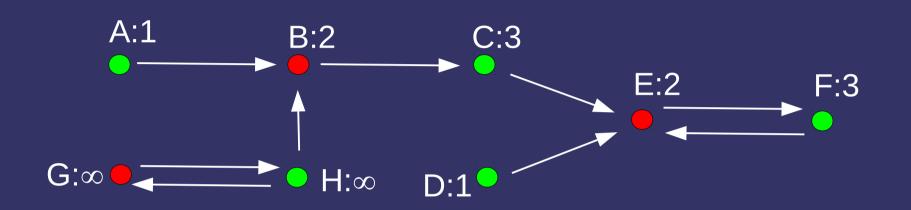


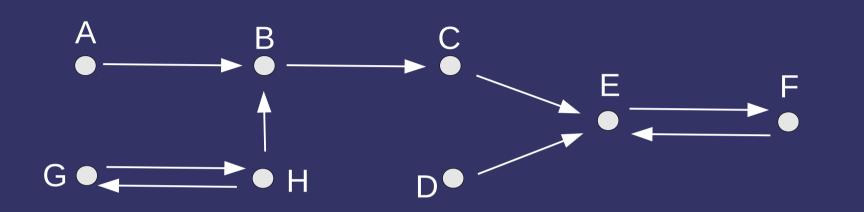


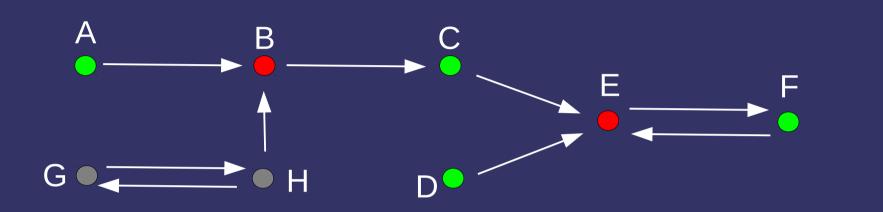


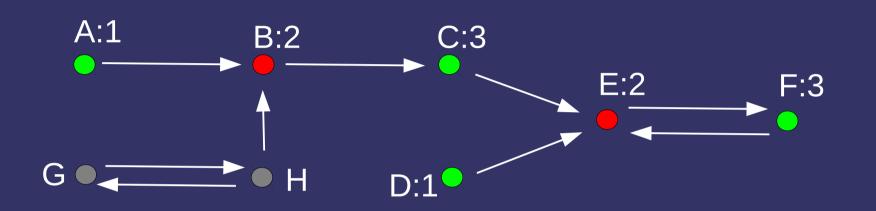












## A Discussion Game for Grounded Semantics (1/3)

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two players:

- proponent (P)
- opponent (O)

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- four moves (A,B ∈ Ar)
  (1) P: HTB(A)
   "A has to be the case"
  (2) O: CB(B)
   "B can be the case"
  (3) O: CONCEDE(A)
   "I now agree that A"
  (4) O: RETRACT(B)
   "I no longer hold that B can be the case"

- two players:
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(2) O: CB(B)

"Perhaps B isn't labelled out by every complete"

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"I now agree that A"

(4) O: RETRACT(B)

"I no longer hold that B can be the case"

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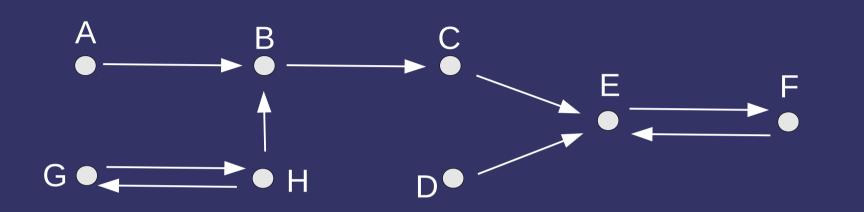
"Perhaps B isn't labelled out by every complete"

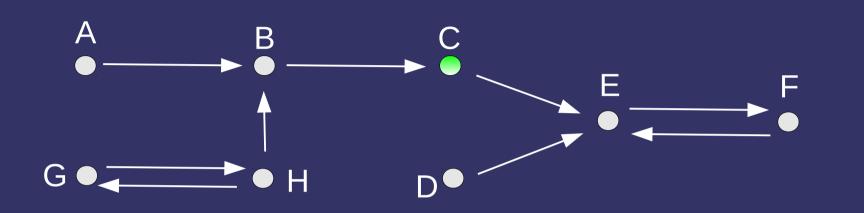
(3) O: CONCEDE(A)

"I now agree that A is labelled in by every complete"

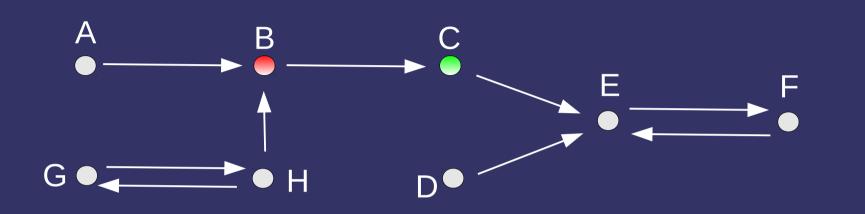
(4) O: RETRACT(B)

"I now think that B is labelled out by every complete"

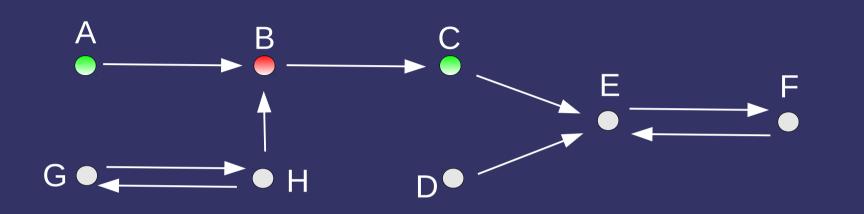




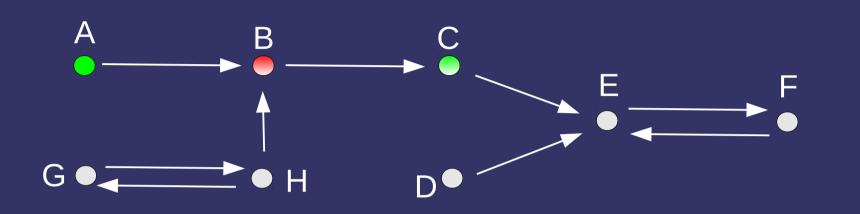
P: HTB(C)



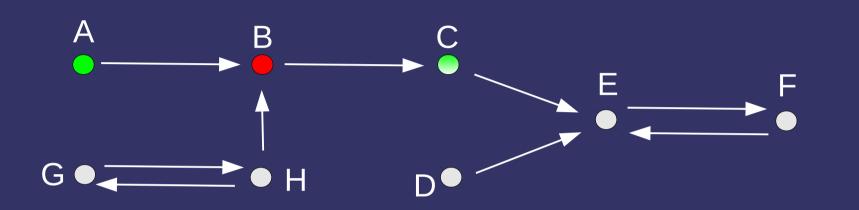
P: HTB(C) O: CB(B)



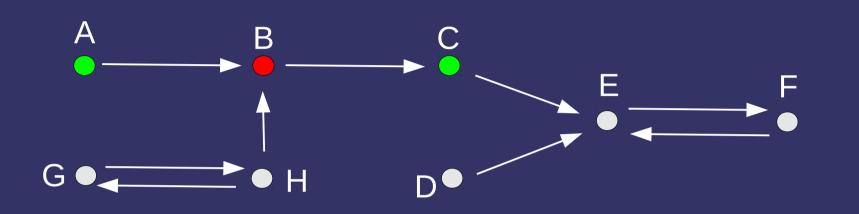
P: HTB(C) O: CB(B) P: HTB(A)



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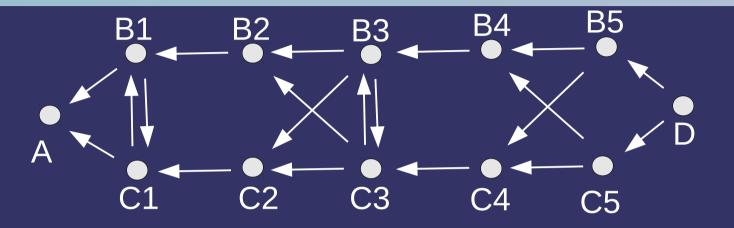
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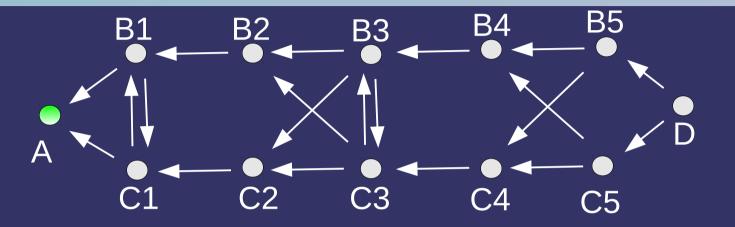
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- RETRACT(B)
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HTB(A) This is either the first move, or

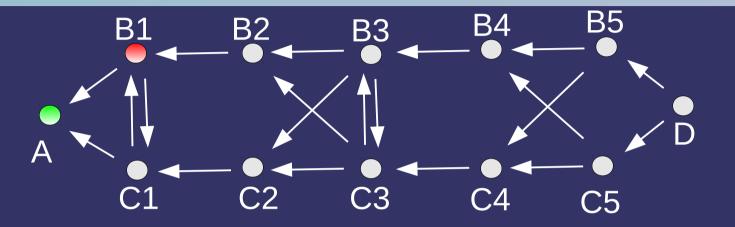
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- there has been a CB(B) statement in the past...
- ...of which an attacker has been CONCEDEd
- and RETRACT(B) hasn't yet been moved

additional condition for all moves no HTB-CB repeats have occurred

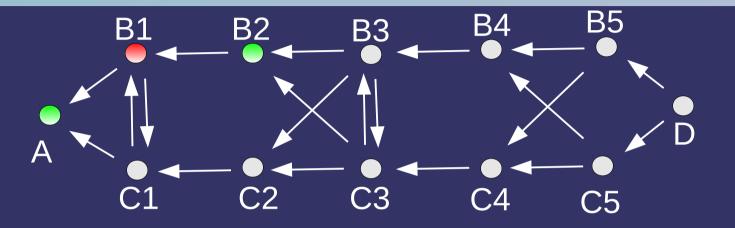




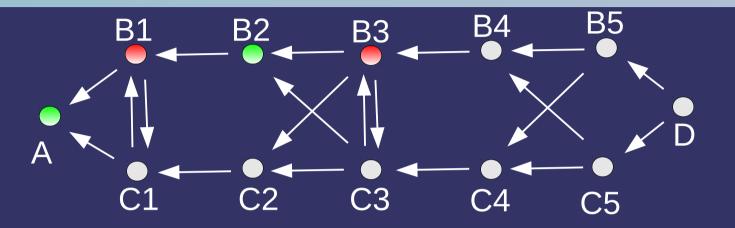
P: HTB(A)



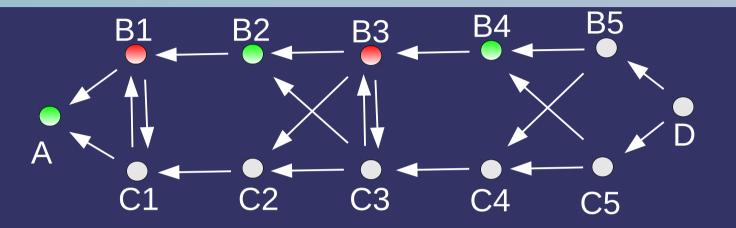
P: HTB(A) O: CB(B1)



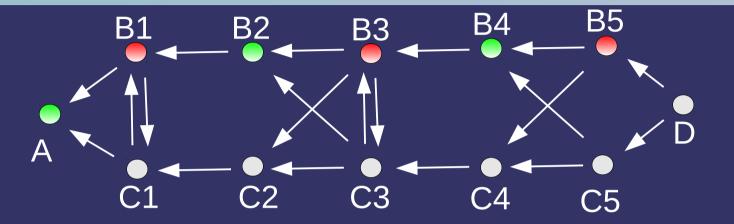
P: HTB(A) O: CB(B1) P: HTB(B2)



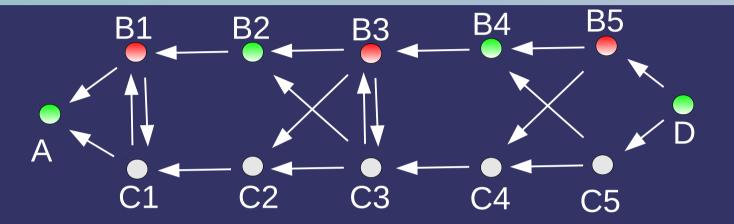
P: HTB(A) O: CB(B1) P: HTB(B2) O: CB(B3)



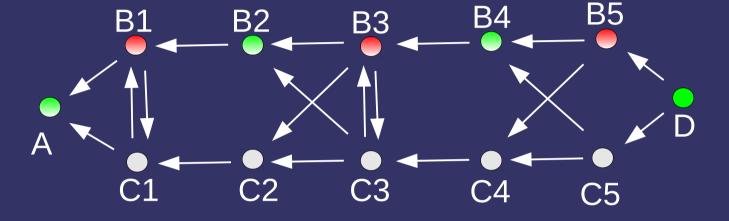
P: HTB(A) O: CB(B1) P: HTB(B2) O: CB(B3) P: HTB(B4)



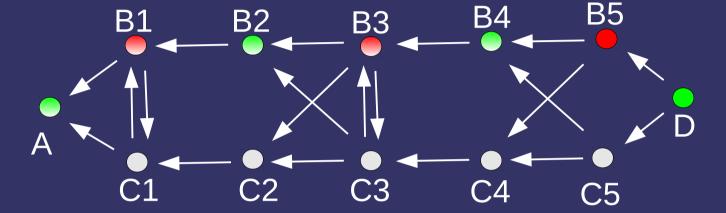
P: HTB(A)
O: CB(B1)
P: HTB(B2)
O: CB(B3)
P: HTB(B4)
O: CB(B5)



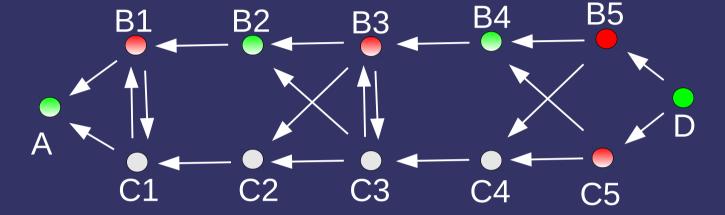
P: HTB(A)
O: CB(B1)
P: HTB(B2)
O: CB(B3)
P: HTB(B4)
O: CB(B5)
P: HTB(D)



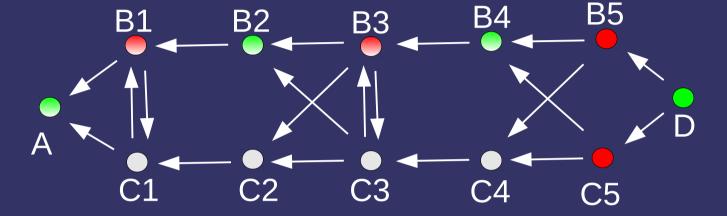
O: RETRACT(B5)



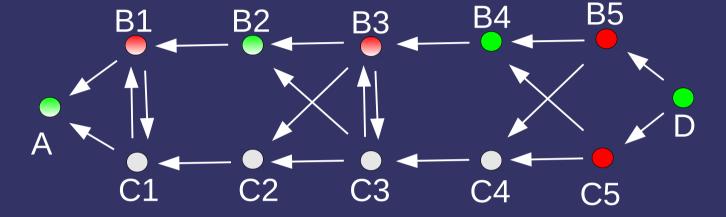
O: RETRACT(B5) O: CB(C5)



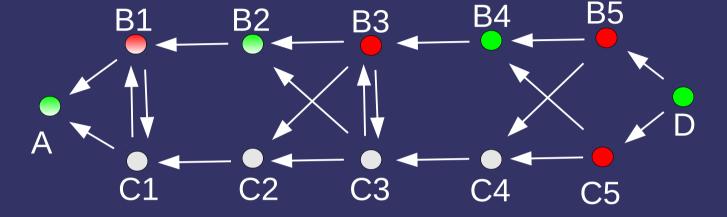
O: RETRACT(B5) O: CB(C5) O: RETRACT(C5)



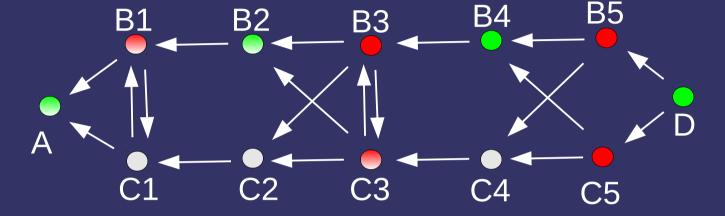
O: RETRACT(B5) O: CB(C5) O: RETRACT(C5) O: CONCEDE(B4)



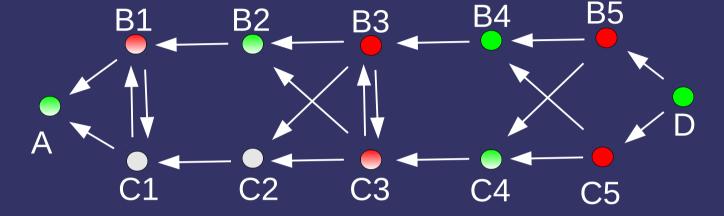
O: RETRACT(B5) O: CB(C5) O: RETRACT(C5) O: CONCEDE(B4) O: RETRACT(B3)



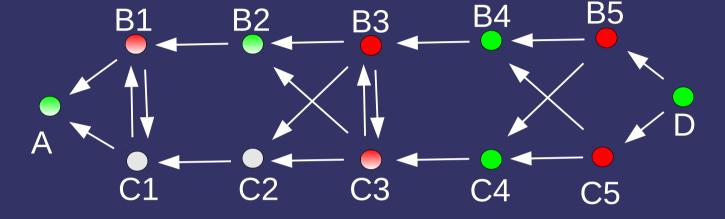
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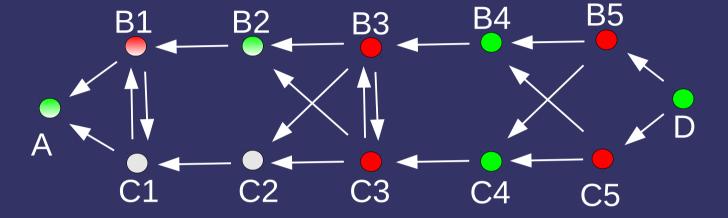
O: RETRACT(B5) O: CB(C5) O: RETRACT(C5) O: CONCEDE(B4) O: RETRACT(B3) O: CB(C3) P: HTB(C4)



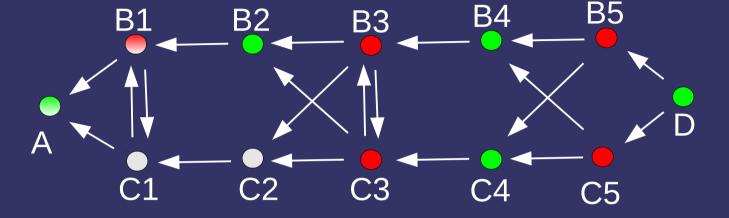
O: RETRACT(B5) O: CB(C5) O: RETRACT(C5) O: CONCEDE(B4) O: RETRACT(B3) O: CB(C3) P: HTB(C4) O: CONCEDE(C4)



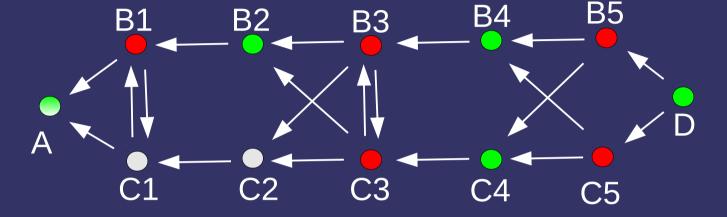
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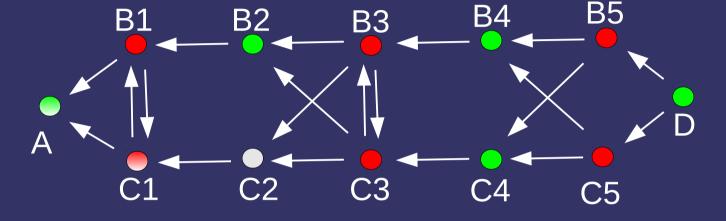
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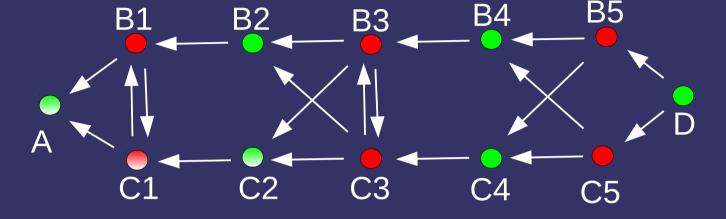
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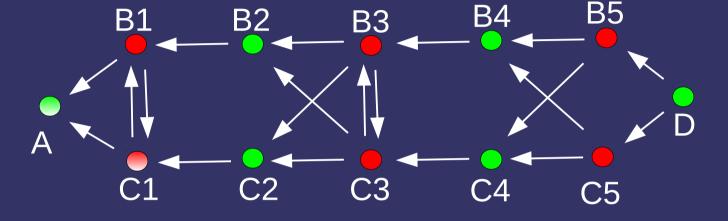
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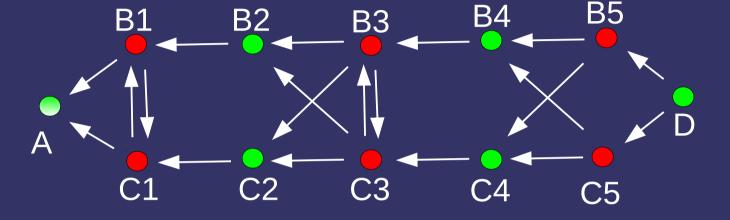
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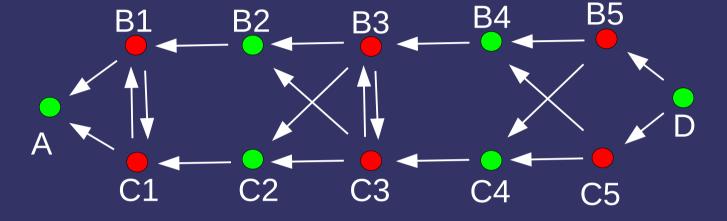


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O: RETRACT(C3) O: CONCEDE(B2) O: RETRACT(B1) O: CB(C1) P: HTB(C2) O: CONCEDE(C2) O: RETRACT(C1) O: CONCEDE(A)











P: HTB(D)





P: HTB(D) O: CB(C)





P: HTB(D) O: CB(C) P: HTB(B)

#### P: HTB(D) O: CB(C) P: HTB(B) O: CB(A)



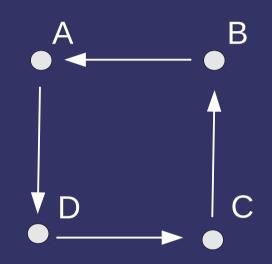


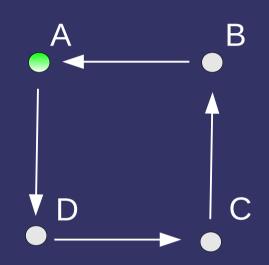




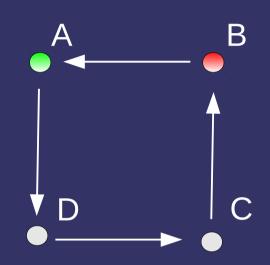
P: HTB(D) O: CB(C) P: HTB(B) O: CB(A)

Proponent cannot move anymore, so discussion is terminated. Main claim not CONCEDEd, so proponent loses.

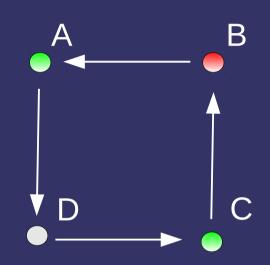






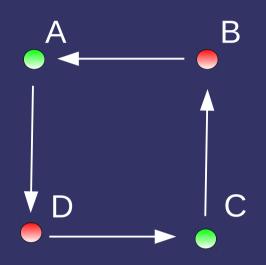


P: HTB(A) O: CB(B)



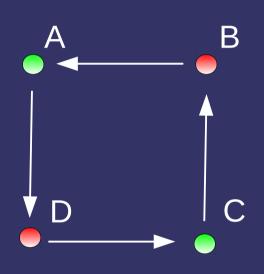
P: HTB(A) O: CB(B) P: HTB(C)

#### P: HTB(A) O: CB(B) P: HTB(C) O: CB(D)

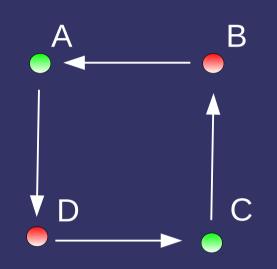




P: HTB(A) O: CB(B) P: HTB(C) O: CB(D) P: HTB(A)







P: HTB(A)
O: CB(B)
P: HTB(C)
O: CB(D)
P: HTB(A)

Now we have a HTB(A)-HTB(A) repeat, so all following moves are blocked: discussion terminated.

> Main claim not CONCEDEd, so proponent loses

• A discussion is *terminated* iff no next move is possible

- A discussion is *terminated* iff no next move is possible
- A terminated discussion (starting with HTB(A)) is won by the proponent iff the opponent has moved CONCEDE(A)

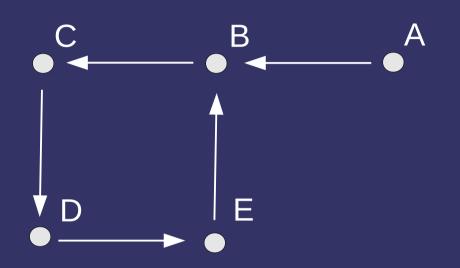
- A discussion is *terminated* iff no next move is possible
- A terminated discussion (starting with HTB(A)) is won by the proponent iff the opponent has moved CONCEDE(A)
- in(Lab<sub>o</sub>) = all CONCEDEd arguments
   out(Lab<sub>o</sub>) = all RETRACTed arguments

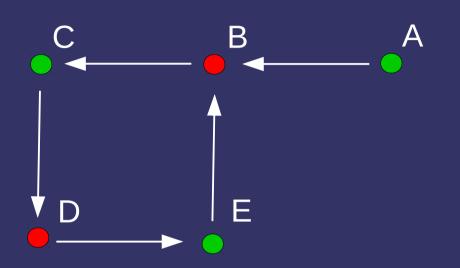
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- in(Lab<sub>o</sub>) = all CONCEDEd arguments
   out(Lab<sub>o</sub>) = all RETRACTed arguments
- At any point of the discussion, Lab<sub>o</sub> is strongly admissible

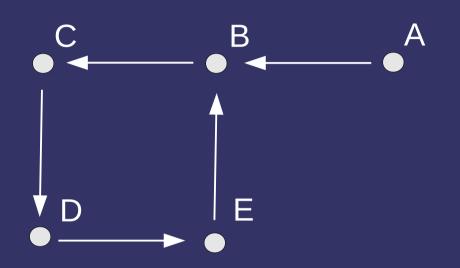
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- <u>completeness</u>: if an argument is in in the grounded labelling, then the proponent has a winning strategy for it

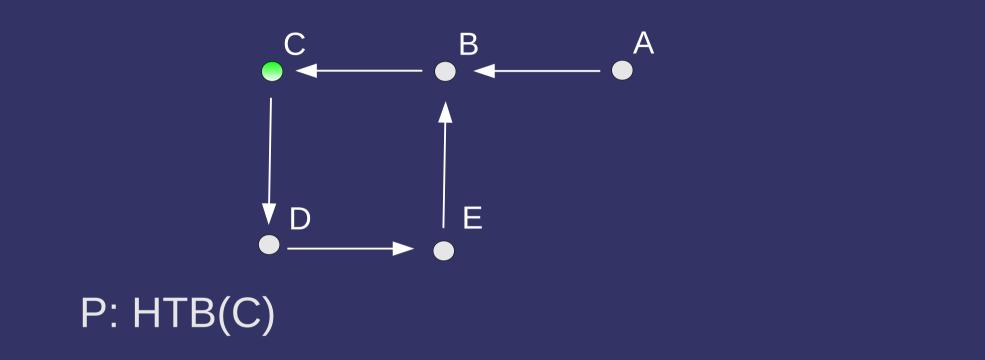
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   out(Lab<sub>o</sub>) = all RETRACTed arguments
- At any point of the discussion, Lab, is strongly admissible
- <u>correctness</u>: if a discussion is won by the proponent, then the main argument is in the grounded extension
- <u>completeness</u>: if an argument is in the grounded extension, then the proponent has a winning strategy for it



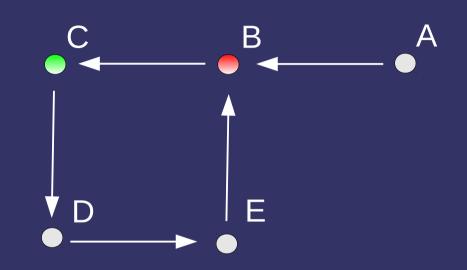






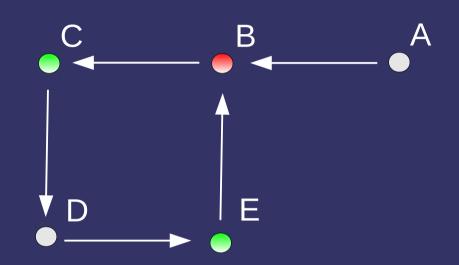






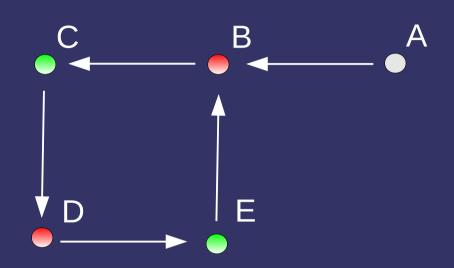
#### P: HTB(C) O: CB(B)





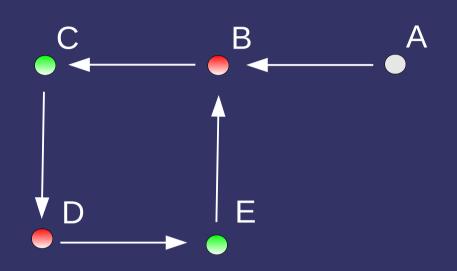
#### P: HTB(C) O: CB(B) P: HTB(E)

#### P: HTB(C) O: CB(B) P: HTB(E) O: CB(D)



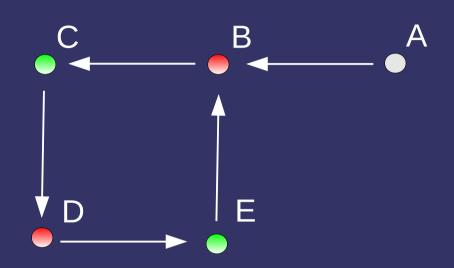


# P: HTB(C) O: CB(B) P: HTB(E) O: CB(D) P: HTB(C)



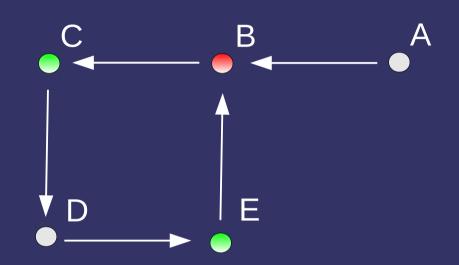


#### P: HTB(C) O: CB(B) P: HTB(E) O: CB(D)



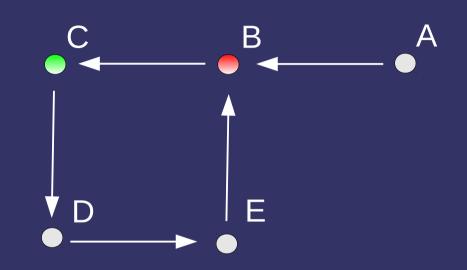






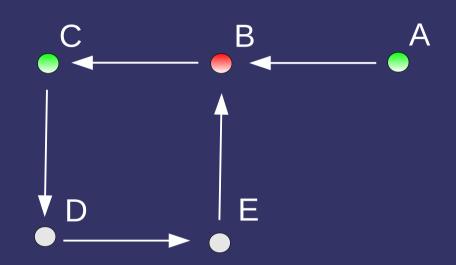
#### P: HTB(C) O: CB(B) P: HTB(E)





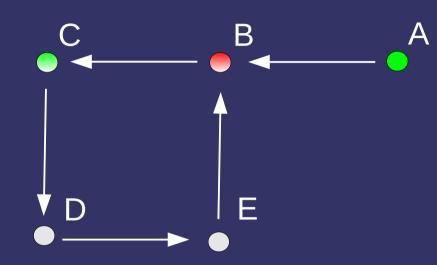
#### P: HTB(C) O: CB(B)





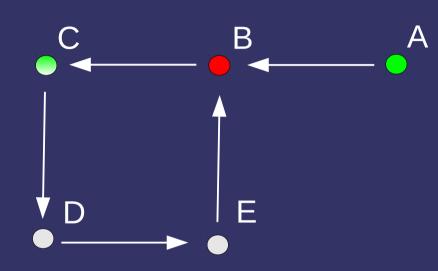
#### P: HTB(C) O: CB(B) P: HTB(A)

#### P: HTB(C) O: CB(B) P: HTB(A) O: CONCEDE(A)



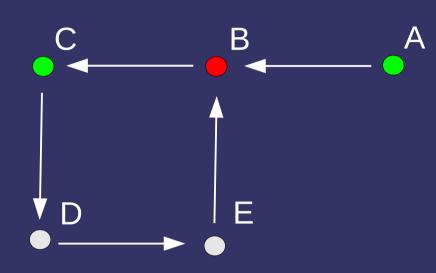


#### P: HTB(C) O: CB(B) P: HTB(A) O: CONCEDE(A) O: RETRACT(B)



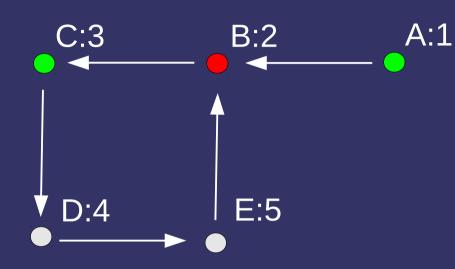


P: HTB(C) O: CB(B) P: HTB(A) O: CONCEDE(A) O: RETRACT(B) O: CONCEDE(C)





P: HTB(C) O: CB(B) P: HTB(A) O: CONCEDE(A) O: RETRACT(B) O: CONCEDE(C)







- Paul: My car is very safe, because it has an airbag.
- Olga: I do not think having an airbag makes your car safe, because airbags are unreliable: the newspapers had several reports on cases where airbags did not work.
- Paul: A recent scientific study shows that airbags are in fact 99% reliable, and scientific studies are more important than newspaper reports.
- Olga: OK, I admit that airbags are reliable. However, your car is not very safe, since its maximum speed is much too high.

- P: My car is very safe, because it has an airbag.
- O: I do not think having an airbag makes your car safe, because airbags are unreliable: the newspapers had several reports on cases where airbags did not work.
- P: A recent scientific study shows that airbags are in fact 99% reliable, and scientific studies are more important than newspaper reports.
- O: OK, I admit that airbags are reliable. However, your car is not very safe, since its maximum speed is much too high.

- P: HTB(airbag  $\Rightarrow$  safe)
- O: I do not think having an airbag makes your car safe, because airbags are unreliable: the newspapers had several reports on cases where airbags did not work.
- P: A recent scientific study shows that airbags are in fact 99% reliable, and scientific studies are more important than newspaper reports.
- O: OK, I admit that airbags are reliable. However, your car is not very safe, since its maximum speed is much too high.

- P: HTB(airbag  $\Rightarrow$  safe)
- O:  $CB(npr \Rightarrow \neg airbagrel \Rightarrow \neg [airbag \Rightarrow safe])$
- P: A recent scientific study shows that airbags are in fact 99% reliable, and scientific studies are more important than newspaper reports.
- O: OK, I admit that airbags are reliable. However, your car is not very safe, since its maximum speed is much too high.

- P: HTB(airbag  $\Rightarrow$  safe)
- O:  $CB(npr \Rightarrow \neg airbagrel \Rightarrow \neg [airbag \Rightarrow safe])$
- P: HTB(study  $\Rightarrow$  airbagrel)
- O: OK, I admit that airbags are reliable. However, your car is not very safe, since its maximum speed is much too high.

- P: HTB(airbag  $\Rightarrow$  safe)
- O:  $CB(npr \Rightarrow \neg airbagrel \Rightarrow \neg [airbag \Rightarrow safe])$
- P: HTB(study  $\Rightarrow$  airbagrel)
- O: CONCEDE(study  $\Rightarrow$  airbagrel) However, your car is not very safe, since its maximum speed is much too high.

- P: HTB(airbag  $\Rightarrow$  safe)
- O:  $CB(npr \Rightarrow \neg airbagrel \Rightarrow \neg [airbag \Rightarrow safe])$
- P: HTB(study  $\Rightarrow$  airbagrel)
- O: CONCEDE(study  $\Rightarrow$  airbagrel) RETRACT(npr  $\Rightarrow \neg$  airbagrel  $\Rightarrow \neg$ [airbag  $\Rightarrow$  safe])

- P: HTB(airbag  $\Rightarrow$  safe)
- O:  $CB(npr \Rightarrow \neg airbagrel \Rightarrow \neg [airbag \Rightarrow safe])$
- P: HTB(study  $\Rightarrow$  airbagrel)
- O: CONCEDE(study  $\Rightarrow$  airbagrel) RETRACT(npr  $\Rightarrow \neg$  airbagrel  $\Rightarrow \neg$ [airbag  $\Rightarrow$  safe]) CB(highspeed  $\Rightarrow \neg$  safe)

#### **Previous Discussion Games**

	Standard Grounded Game	Grounded Persuasion Game	Grounded Discussion Game
#moves needed to show str. adm.		linear	linear
RETRACT/ CONCEDE moves		yes	yes
single successful game is sufficient		yes	yes
in grounded implies ∃ winning strategy	yes		yes