

Open Research Challenges in Formal Argumentation Theory

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We want to express entailment as dialectics

We want to express formal argument-based entailment as the ability to win a discussion.

This is because:

- formal logic started as dialectics
- it is in line with the nature of symbolic AI
- it follows the tradition of Lorenz & Lorenzen
- foundational problem: truth and model-based semantics not suitable for NMR
- we need to go towards applications; explainability is key

Requirements

What we want is an argument game (=dialectical protocol) that:

- works with rule-based arguments
- has intuitive (user acceptable) rules regarding the allowed discussion moves
- is sound: if the proponent wins the discussion, then the main argument is accepted w.r.t. formal argumentation theory
- is complete: if the main argument is accepted w.r.t. formal argumentation theory, then the proponent has a winning strategy

Argumentation Theory: How to Construct Arguments

strict rule (\rightarrow):

“from ... it always follows that...”

defeasible rule (\Rightarrow):

“from ... it usually follows that...”

$$R_s = \{ \rightarrow a; a \rightarrow \neg b; \rightarrow d; d \rightarrow e \}$$

$$R_d = \{ \neg b \Rightarrow c; e \Rightarrow \neg c \}$$

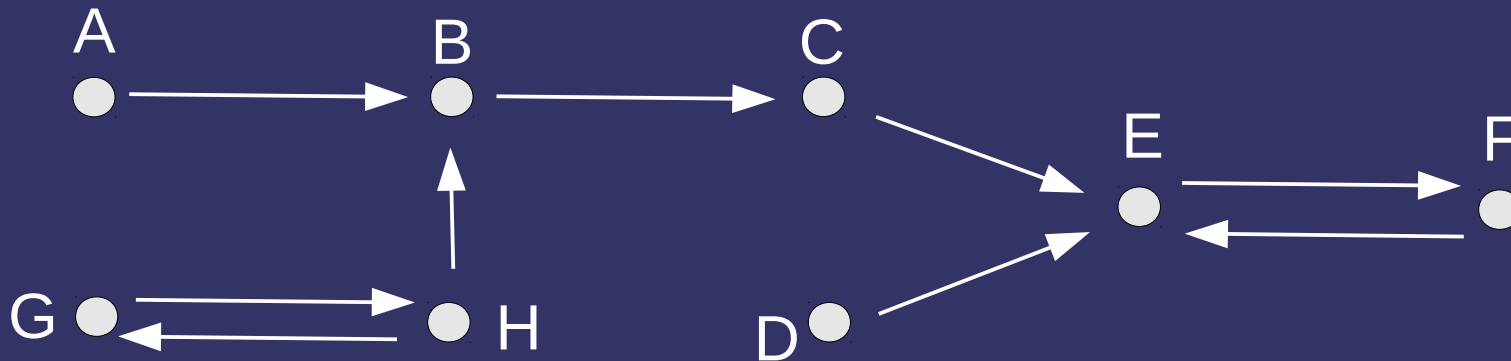
$$A_1: ((\rightarrow a) \rightarrow \neg b) \Rightarrow c$$

$$A_2: ((\rightarrow d) \rightarrow e) \Rightarrow \neg c$$

Argumentation Theory: How to Evaluate Arguments

complete labelling:

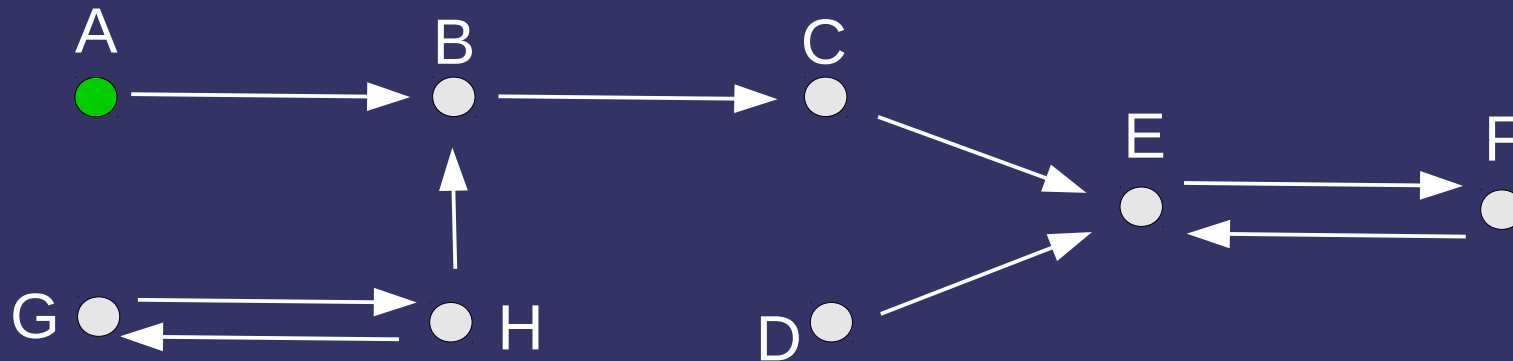
- **in** \Leftrightarrow all attackers **out**
- **out** \Leftrightarrow there is an attacker **in**
- **undec** \Leftrightarrow not all attackers **out** and no attacker **in**



Argumentation Theory: How to Evaluate Arguments

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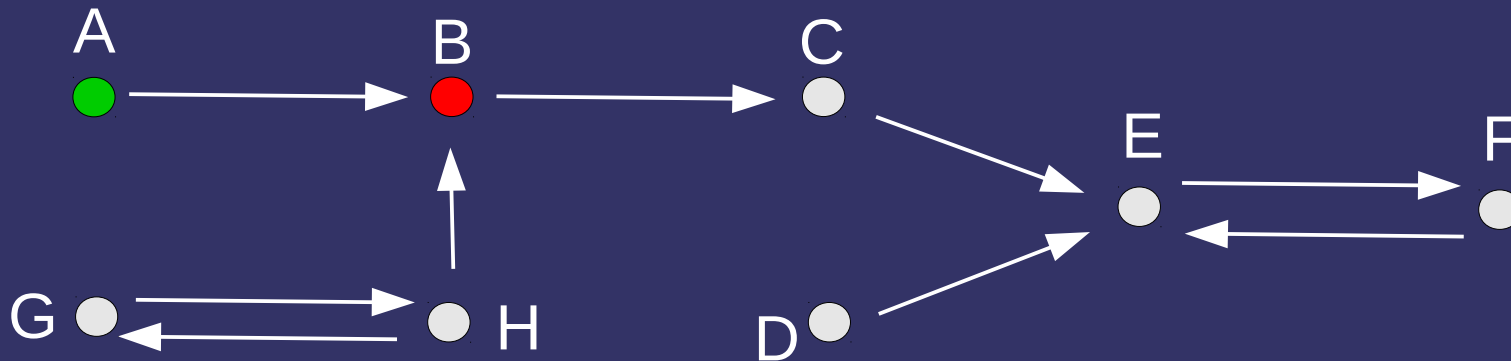
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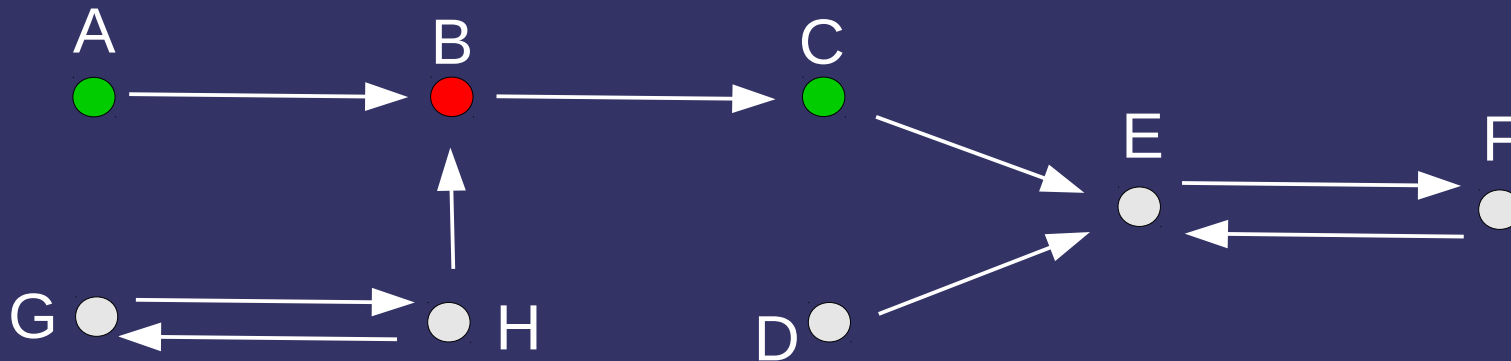
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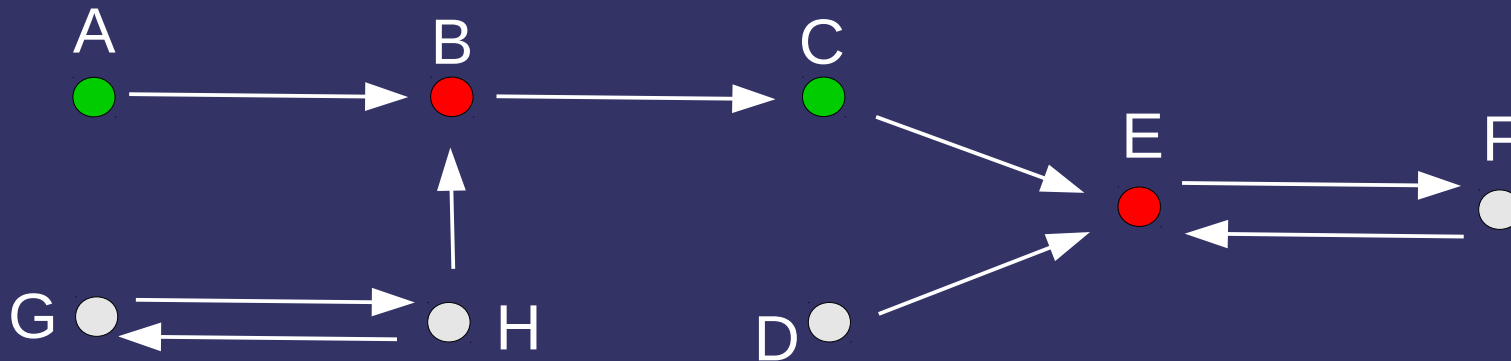
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Argumentation Theory: How to Evaluate Arguments

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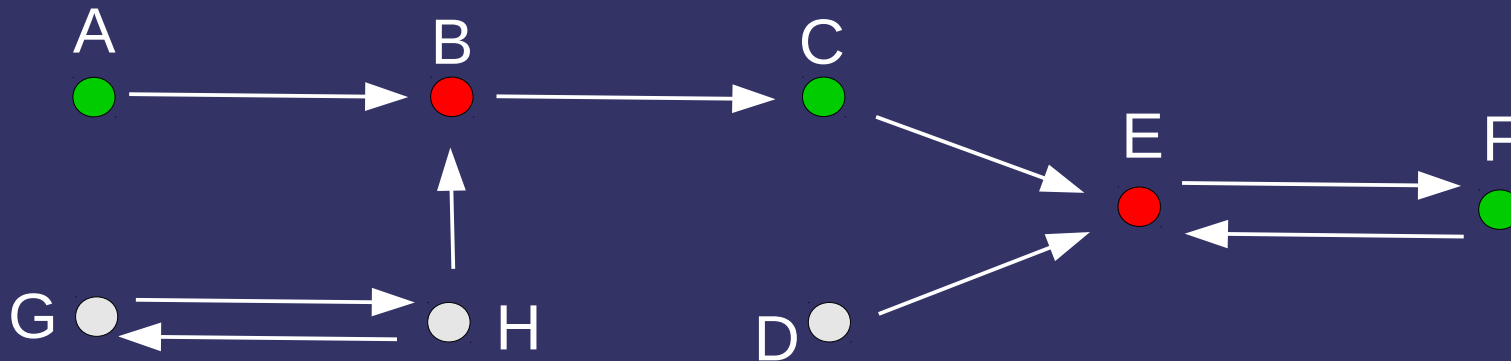
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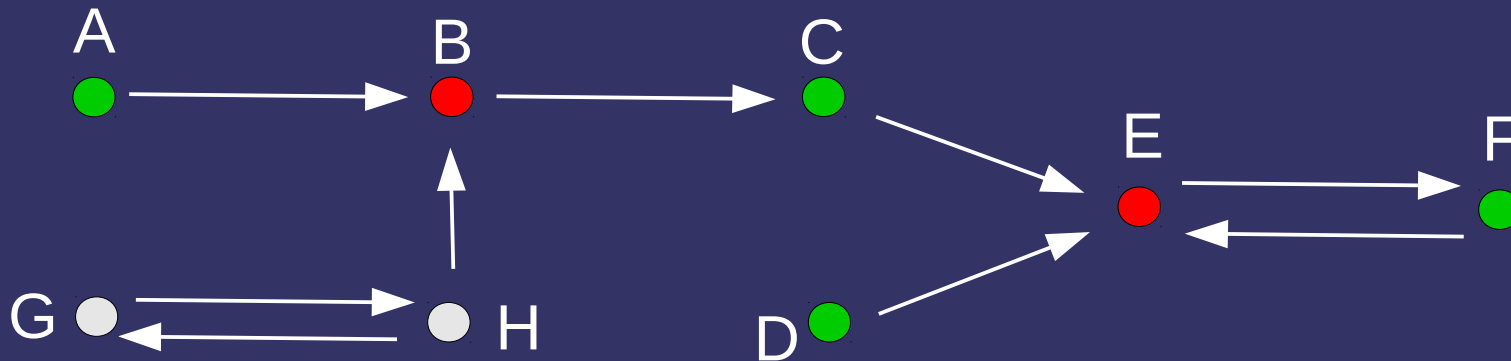
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Argumentation Theory: How to Evaluate Arguments

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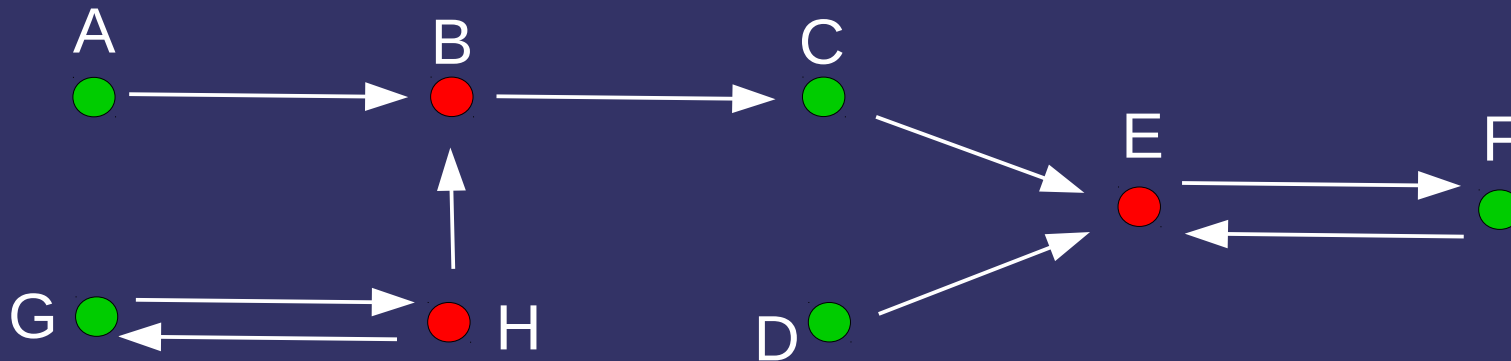
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Argumentation Theory: How to Evaluate Arguments

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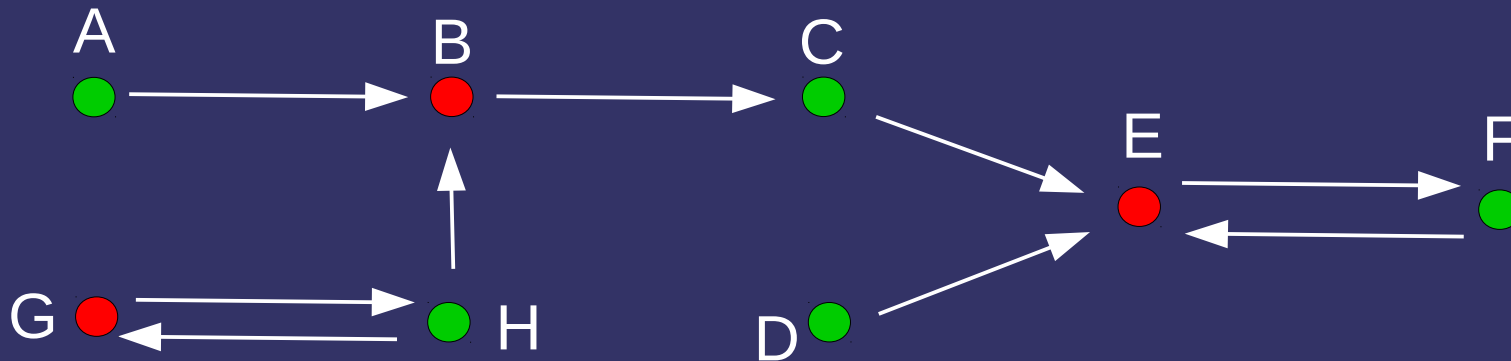
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Argumentation Theory: How to Evaluate Arguments

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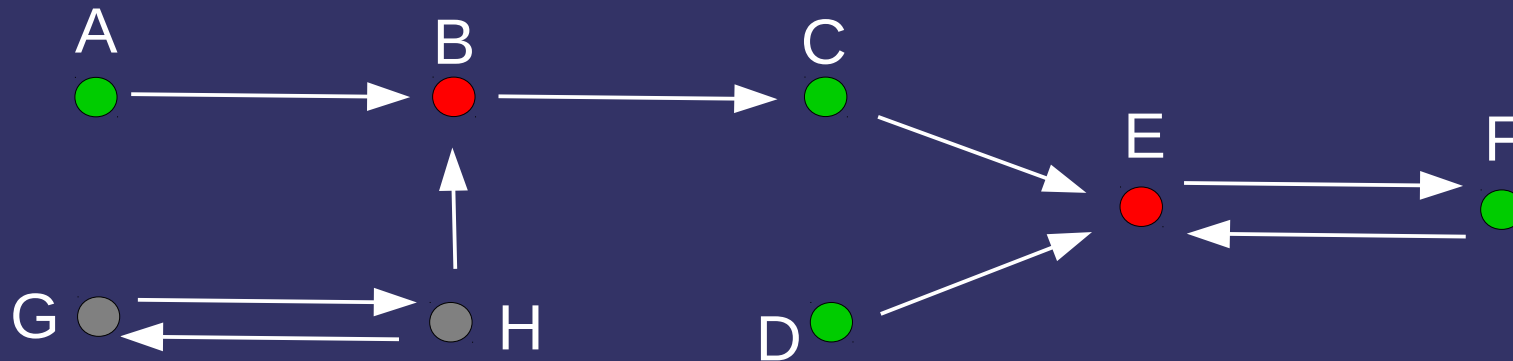
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Argumentation Theory: How to Evaluate Arguments

complete labelling:

- **in** \Leftrightarrow all attackers **out**
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Argumentation Theory: How to Evaluate Arguments

complete labelling:

- **in** \Leftrightarrow all attackers **out**
- **out** \Leftrightarrow there is an attacker **in**
- **undec** \Leftrightarrow not all attackers **out** and no attacker **in**

preferred labelling:

complete labelling with maximal **in**
(= complete labelling with maximal **out**)

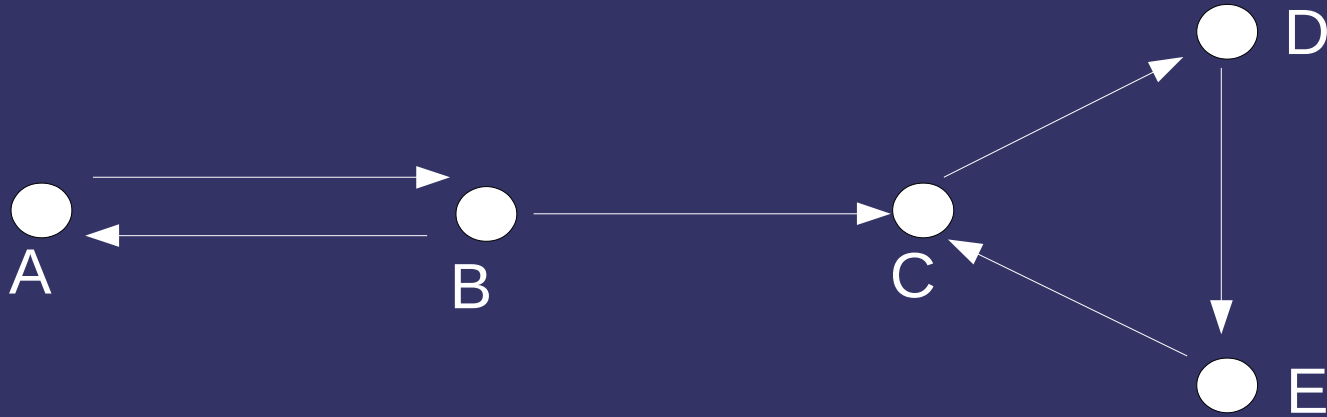
grounded labelling:

complete labelling with maximal **undec**
(= complete labelling with minimal **in/out**)

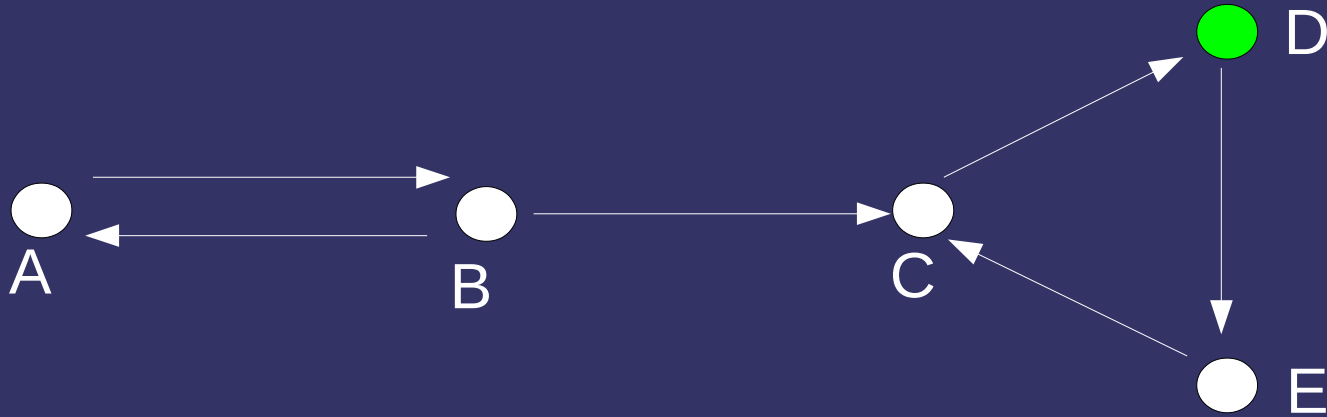
The Good News

- sound and complete discussion games have been stated for many types of labellings:
 - complete
 - preferred
 - grounded
 - stable
 - ideal
- We will now give an example of a discussion game for complete labellings...

Discussion Game for Complete Semantics

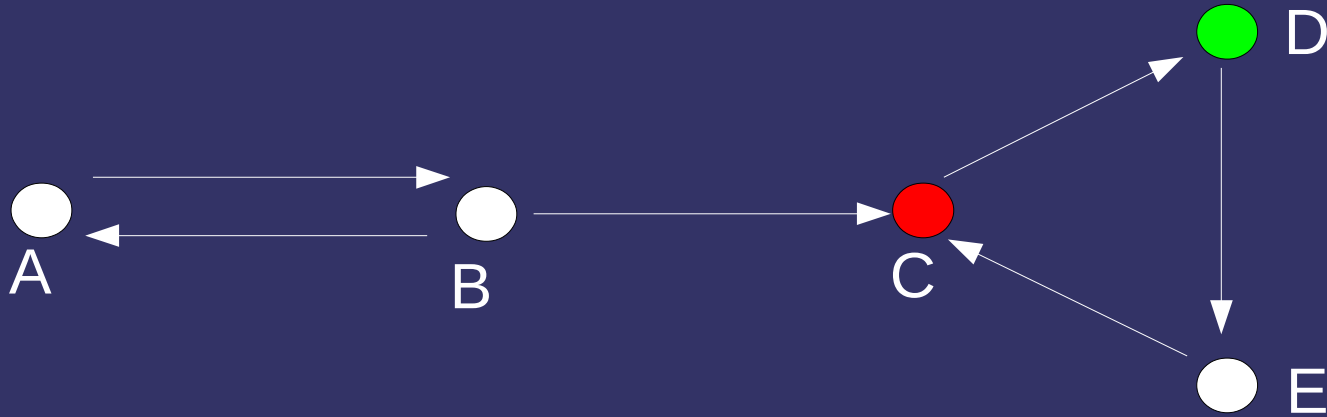


Discussion Game for Complete Semantics



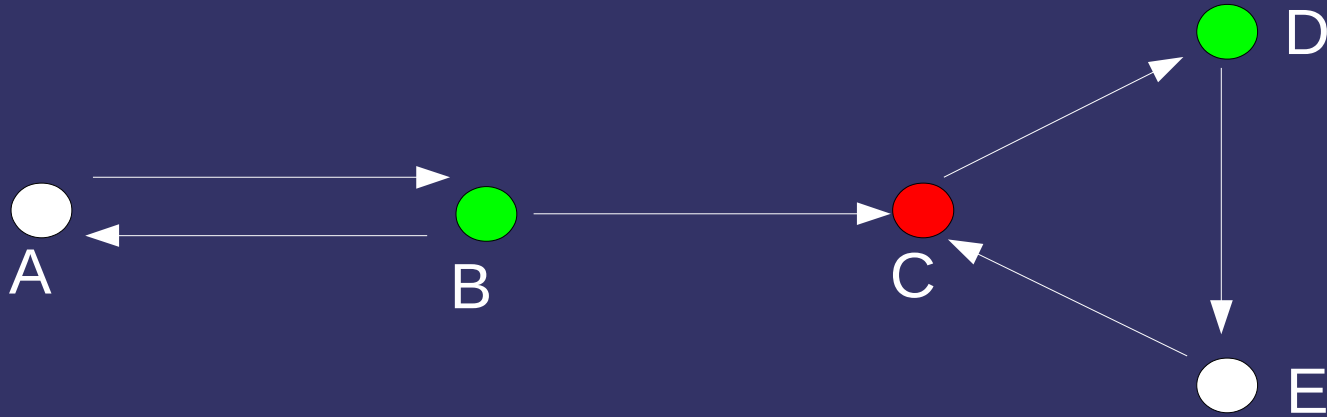
M: $\text{in}(D)$ “I have a complete labelling in which D is labelled in .”

Discussion Game for Complete Semantics



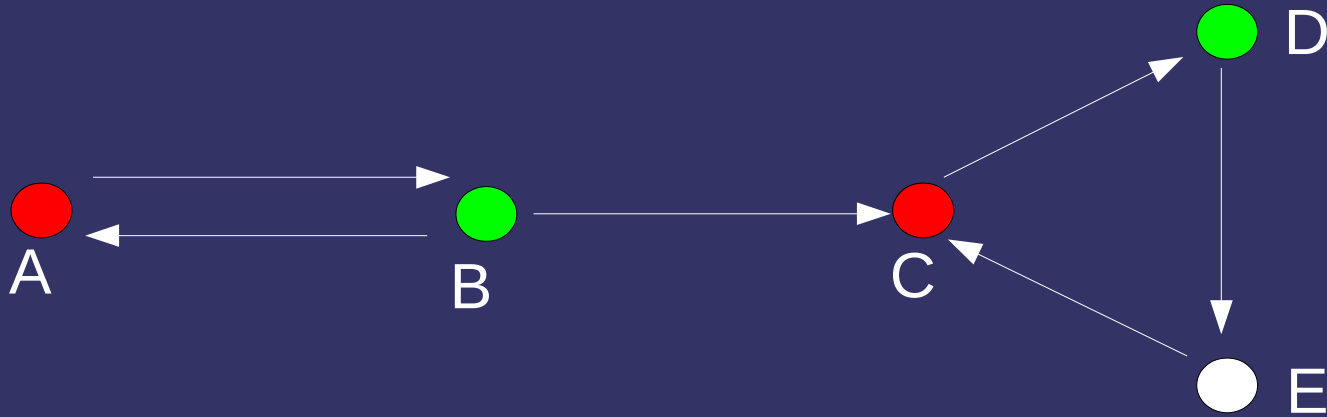
M: **in**(D) “I have a complete labelling in which D is labelled **in**.”
S: **out**(C) “But then in your labelling it must also be the case that D’s attacker C is labelled **out**. Based on which grounds?”

Discussion Game for Complete Semantics



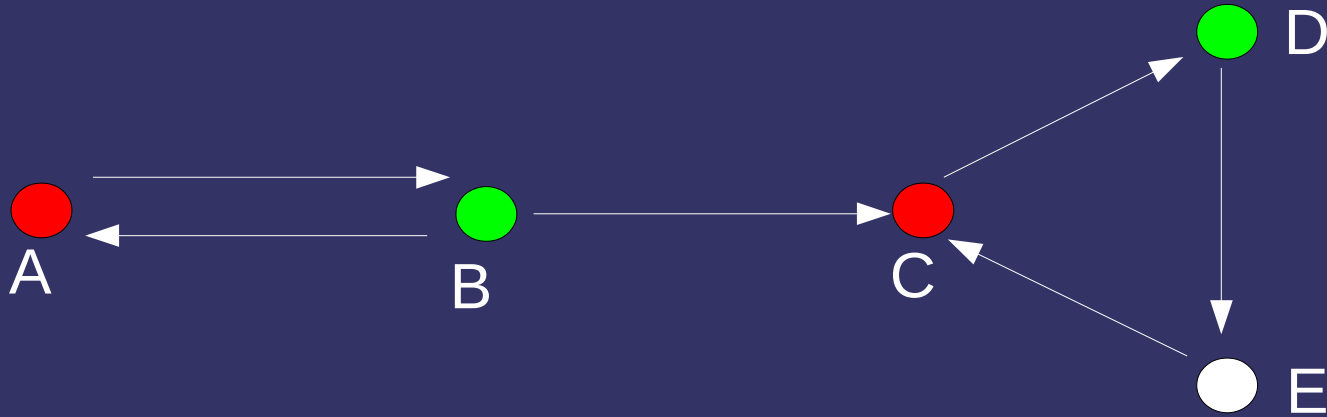
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- M: **in**(B) “C is labelled **out** because B is labelled **in**.”

Discussion Game for Complete Semantics



- M: **in**(D) “I have a complete labelling in which D is labelled **in**.”
- S: **out**(C) “But then in your labelling it must also be the case that D’s attacker C is labelled **out**. Based on which grounds?”
- M: **in**(B) “C is labelled **out** because B is labelled **in**.”
- S: **out**(A) “But then in your labelling it must also be the case that B’s attacker A is labelled **out**. Based on which grounds?”

Discussion Game for Complete Semantics



- M: **in**(D) “I have a complete labelling in which D is labelled **in**.”
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- M: **in**(B) “A is labelled **out** because B is labelled **in**.”

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- M: **in**(D) “I have a complete labelling in which D is labelled **in**.”
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Discussion Game for Complete Semantics

M: **in**(D) “I have a complete labelling in which D is labelled **in**.”

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(1) Each move of M (except the first) contains an attacker of the directly preceding move of S.

Discussion Game for Complete Semantics

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- S: **out**(C) “But then in your labelling it must also be the case that D’s attacker C is labelled **out**. Based on which grounds?”
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*(2) Each move of S
contains an attacker of some previous move of M.*

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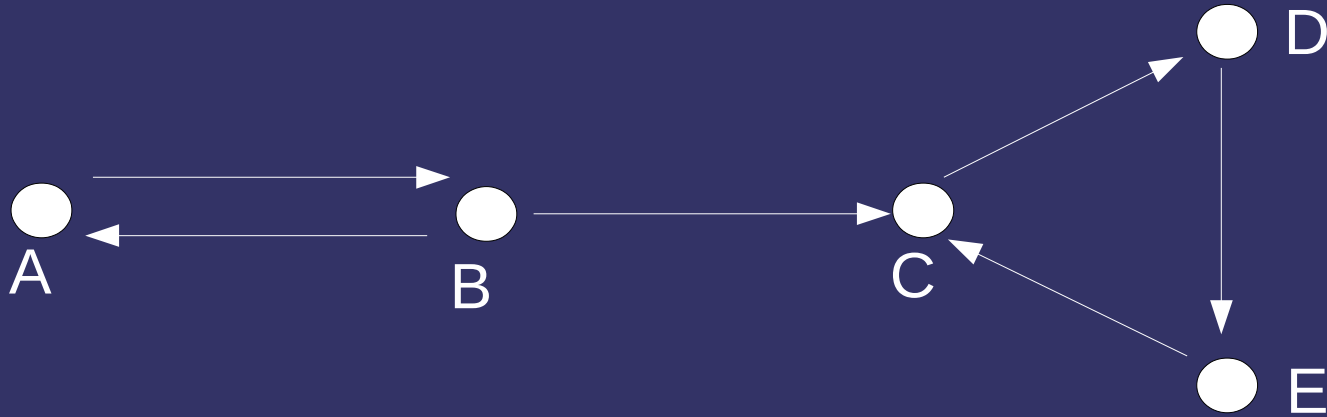
(3) S is not allowed to repeat his moves.

Discussion Game for Complete Semantics

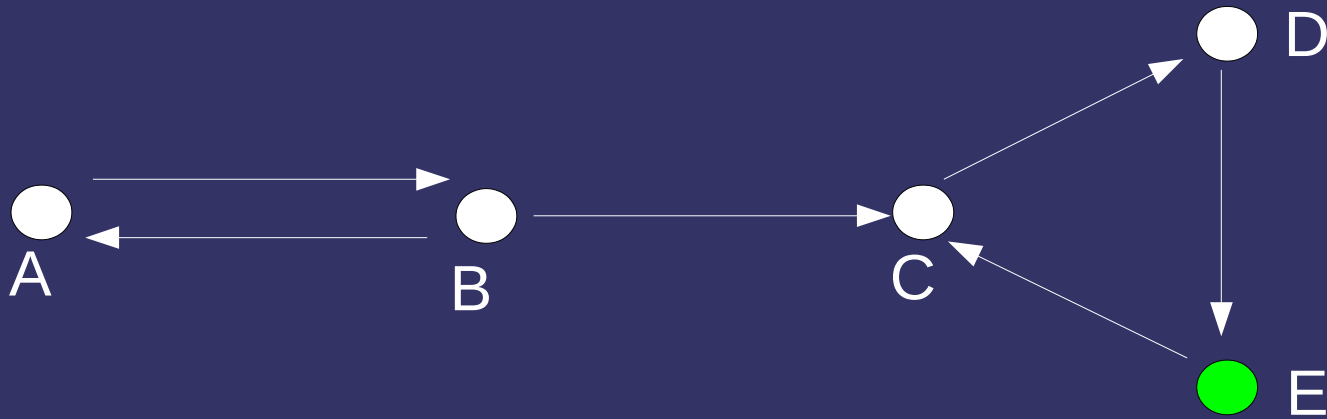
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(4) M is allowed to repeat his moves.

Discussion Game for Complete Semantics

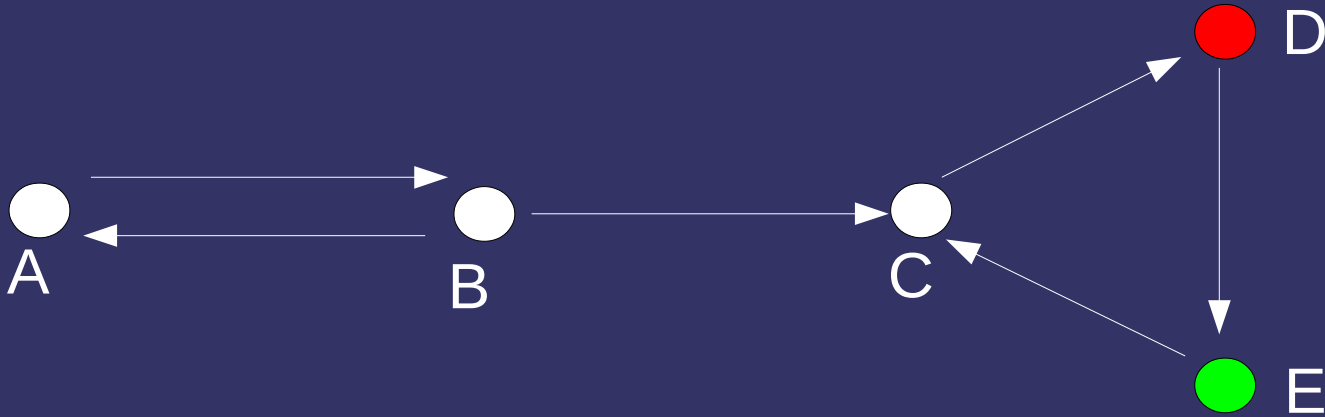


Discussion Game for Complete Semantics



M: $in(E)$ “I have a complete labelling in which E is labelled in .”

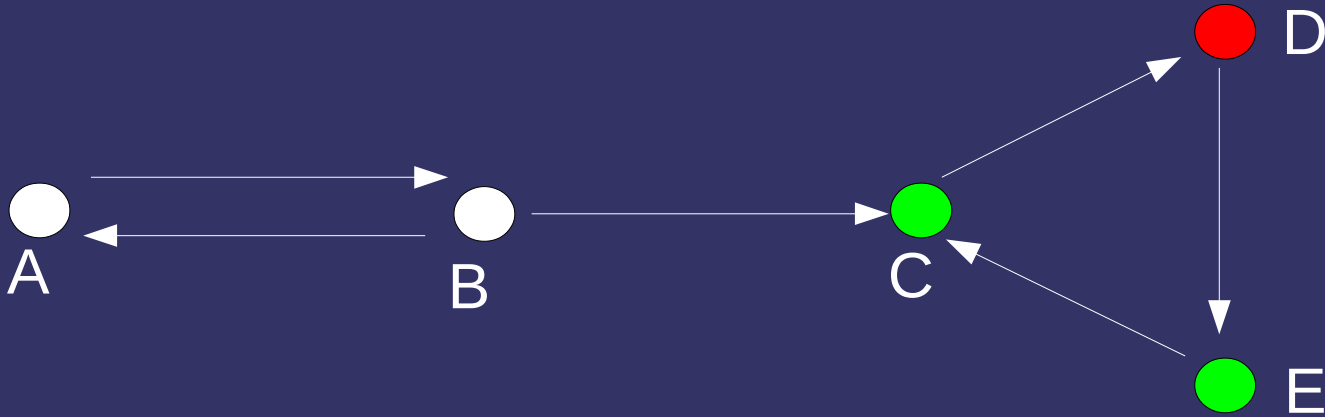
Discussion Game for Complete Semantics



M: **in**(E) “I have a complete labelling in which E is labelled **in**.”

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Discussion Game for Complete Semantics

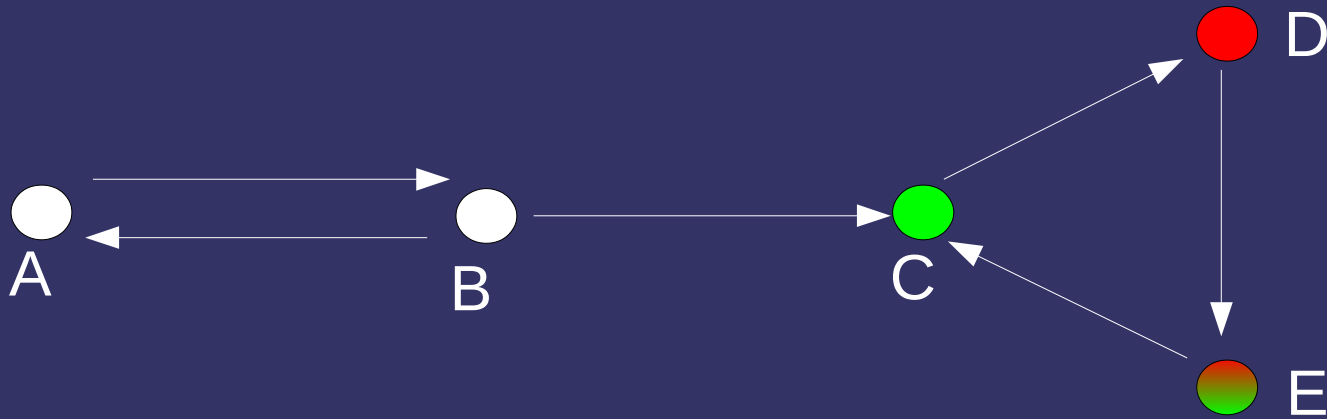


M: **in**(E) “I have a complete labelling in which E is labelled **in**.”

S: **out**(D) “But then in your labelling it must also be the case that E's attacker D is labelled **out**. Based on which grounds?”

M: **in**(C) “D is labelled **out** because C is labelled **in**.”

Discussion Game for Complete Semantics



- M: **in**(E) “I have a complete labelling in which E is labelled **in**.”
- S: **out**(D) “But then in your labelling it must also be the case that E's attacker D is labelled **out**. Based on which grounds?”
- M: **in**(C) “D is labelled **out** because C is labelled **in**.”
- S: **out**(E) “But then in your labelling it must also be the case that C's attacker E is labelled **out**. This contradicts with your earlier claim that E is labelled **in**.”

Discussion Game for Complete Semantics

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(5) If S uses an argument previously used by M, then S wins the discussion.

Discussion Game for Complete Semantics

- M: **in**(E) “I have a complete labelling in which E is labelled **in**.”
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- M: **in**(C) “D is labelled **out** because C is labelled **in**.”
- S: **out**(E) “But then in your labelling it must also be the case that C's attacker E is labelled **out**. This contradicts with your earlier claim that E is labelled **in**.”

(6) If M uses an argument previously used by S, then S wins the discussion.

Discussion Game for Complete Semantics

M: **in**(E) “I have a complete labelling in which E is labelled **in**.”

S: **out**(D) “But then in your labelling it must also be the case that E's attacker D is labelled **out**. Based on which grounds?”

M: **in**(C) “D is labelled **out** because C is labelled **in**.”

S: **out**(E) “But then in your labelling it must also be the case that C's attacker E is labelled **out**. This contradicts with your earlier claim that E is labelled **in**.”

*(7) If M cannot make a move anymore,
then S wins the discussion.*

Discussion Game for Complete Semantics

- M: **in**(E) “I have a complete labelling in which E is labelled **in**.”
- S: **out**(D) “But then in your labelling it must also be the case that E's attacker D is labelled **out**. Based on which grounds?”
- M: **in**(C) “D is labelled **out** because C is labelled **in**.”
- S: **out**(E) “But then in your labelling it must also be the case that C's attacker E is labelled **out**. This contradicts with your earlier claim that E is labelled **in**.”

*(8) If S cannot make a move anymore,
then M wins the discussion.*

Discussion Game for Complete Semantics

- (1) Each move of M (except the first) contains an attacker of the directly preceding move of S.*
- (2) Each move of S contains an attacker of some previous move of M.*
- (3) S is not allowed to repeat his moves.*
- (4) M is allowed to repeat his moves.*
- (5) If S uses an argument previously used by M, then S wins the discussion.*
- (6) If M uses an argument previously used by S, then S wins the discussion.*
- (7) If M cannot make a move anymore, then S wins the discussion.*
- (8) If S cannot make a move anymore, then M wins the discussion.*

Constructing the Graph

What Attacks What

$((d) \Rightarrow e) \Rightarrow \neg c$

$((a) \Rightarrow b) \Rightarrow c$

strict rule (\rightarrow):

“from ... it always follows that...”

defeasible rule (\Rightarrow):

“from ... it usually follows that...”

Constructing the Graph

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Constructing the Graph

What Attacks What

$((d) \rightarrow e) \Rightarrow \neg c$

$((a) \Rightarrow b) \rightarrow c$

strict rule (\rightarrow):

“from ... it always follows that...”

defeasible rule (\Rightarrow):

“from ... it usually follows that...”

Some Terminology

- unrestricted rebut: *check if any previous rule (of the attacked conclusion) is defeasible*

so $((d) \rightarrow e) \Rightarrow \neg c$ attacks

$((a) \Rightarrow b) \rightarrow c$

but not $((a) \rightarrow b) \rightarrow c$

- restricted rebut: *check if the last rule (of the attacked conclusion) is defeasible*

so $((d) \rightarrow e) \Rightarrow \neg c$ attacks

$((a) \rightarrow b) \Rightarrow c$

but not $((a) \Rightarrow b) \rightarrow c$

A Sample Discussion

John: *“Bob will go to both AAMAS and IJCAI this year,
as he has papers accepted at each of these conferences.”*

Mary: *“That won't be possible,
as his budget only allows one foreign trip.”*

A Sample Discussion

$S = \{ \rightarrow\text{paperA}; \rightarrow\text{paperI}; \rightarrow\text{budget};$

$D = \{ \text{paperA} \Rightarrow \text{goA}; \text{paperI} \Rightarrow \text{goI}; \text{budget} \Rightarrow \neg \text{goBoth} \}$

John: *“Bob will go to both AAMAS and IJCAI this year,
as he has papers accepted at each of these conferences.”*

Mary: *“That won't be possible,
as his budget only allows one foreign trip.”*

A Sample Discussion

$S = \{ \rightarrow \text{paperA}; \rightarrow \text{paperI}; \rightarrow \text{budget};$
 $\text{goA}, \text{goI} \rightarrow \text{goboth}; \text{goA}, \neg \text{goboth} \rightarrow \neg \text{goI}; \text{goI}, \neg \text{goboth} \rightarrow \neg \text{goA} \}$
 $D = \{ \text{paperA} \Rightarrow \text{goA}; \text{paperI} \Rightarrow \text{goI}; \text{budget} \Rightarrow \neg \text{goboth} \}$

John: *“Bob will go to both AAMAS and IJCAI this year,
as he has papers accepted at each of these conferences.”*

Mary: *“That won't be possible,
as his budget only allows one foreign trip.”*

A Sample Discussion

$S = \{ \neg \text{paperA}; \neg \text{paperI}; \neg \text{budget};$
 $\text{goA}, \text{goI} \rightarrow \text{goboth}; \text{goA}, \neg \text{goboth} \rightarrow \neg \text{goI}; \text{goI}, \neg \text{goboth} \rightarrow \neg \text{goA} \}$
 $D = \{ \text{paperA} \Rightarrow \text{goA}; \text{paperI} \Rightarrow \text{goI}; \text{budget} \Rightarrow \neg \text{goboth} \}$

John: $((\neg \text{paperA}) \Rightarrow \text{goA}), ((\neg \text{paperI}) \Rightarrow \text{goI}) \rightarrow \text{goboth}$

Mary: $(\neg \text{budget}) \Rightarrow \neg \text{goboth}$

John: *“Bob will go to both AAMAS and IJCAI this year,
as he has papers accepted at each of these conferences.”*

Mary: *“That won't be possible,
as his budget only allows one foreign trip.”*

A Sample Discussion

$S = \{ \rightarrow\text{paperA}; \rightarrow\text{paperI}; \rightarrow\text{budget};$
 $\text{goA}, \text{goI} \rightarrow \text{goboth}; \text{goA}, \neg\text{goboth} \rightarrow \neg\text{goI}; \text{goI}, \neg\text{goboth} \rightarrow \neg\text{goA} \}$
 $D = \{ \text{paperA} \Rightarrow \text{goA}; \text{paperI} \Rightarrow \text{goI}; \text{budget} \Rightarrow \neg\text{goboth} \}$

John: $((\rightarrow\text{paperA}) \Rightarrow \text{goA}), ((\rightarrow\text{paperI}) \Rightarrow \text{goI}) \rightarrow \text{goboth}$

Mary: $((\rightarrow\text{paperA}) \Rightarrow \text{goA}), ((\rightarrow\text{budget}) \Rightarrow \neg\text{goboth}) \rightarrow \neg\text{goI}$

John: *“Bob will go to both AAMAS and IJCAI this year,
as he has papers accepted at both conferences.”*

Mary: *“Bob goes to AAMAS, so he cannot go to IJCAI,
as his budget only allows one foreign trip.”*

A Sample Discussion

$S = \{ \rightarrow\text{paperA}; \rightarrow\text{paperI}; \rightarrow\text{budget};$
 $\text{goA}, \text{goI} \rightarrow \text{goboth}; \text{goA}, \neg\text{goboth} \rightarrow \neg\text{goI}; \text{goI}, \neg\text{goboth} \rightarrow \neg\text{goA} \}$
 $D = \{ \text{paperA} \Rightarrow \text{goA}; \text{paperI} \Rightarrow \text{goI}; \text{budget} \Rightarrow \neg\text{goboth} \}$

John: $((\rightarrow\text{paperA}) \Rightarrow \text{goA}), ((\rightarrow\text{paperI}) \Rightarrow \text{goI}) \rightarrow \text{goboth}$

Mary: $((\rightarrow\text{paperA}) \Rightarrow \text{goI}), ((\rightarrow\text{budget}) \Rightarrow \neg\text{goboth}) \rightarrow \neg\text{goA}$

John: *“Bob will go to both AAMAS and IJCAI this year,
as he has papers accepted at both conferences.”*

Mary: *“Bob goes to IJCAI, so he cannot go to AAMAS,
as his budget only allows one foreign trip.”*

Why Restricted Rebut

$S = \{ w1; w2; w3; \}$

$D = \{ w1 \Rightarrow b1; w2 \Rightarrow b2; w3 \Rightarrow b3 \}$

Why Restricted Rebut

$S = \{ w1; w2; w3; b2, b3 \rightarrow \neg b1; b1, b3 \rightarrow \neg b2; b1, b2 \rightarrow \neg b3 \}$

$D = \{ w1 \Rightarrow b1; w2 \Rightarrow b2; w3 \Rightarrow b3 \}$

Why Restricted Rebut

$S = \{ w1; w2; w3; b2, b3 \rightarrow \neg b1; b1, b3 \rightarrow \neg b2; b1, b2 \rightarrow \neg b3 \}$

$D = \{ w1 \Rightarrow b1; w2 \Rightarrow b2; w3 \Rightarrow b3 \}$

$A_1: (w1) \Rightarrow b1$

$A_2: (w2) \Rightarrow b2$

$A_3: (w3) \Rightarrow b3$

Why Restricted Rebut

$S = \{ w1; w2; w3; b2, b3 \rightarrow \neg b1; b1, b3 \rightarrow \neg b2; b1, b2 \rightarrow \neg b3 \}$

$D = \{ w1 \Rightarrow b1; w2 \Rightarrow b2; w3 \Rightarrow b3 \}$

$A_1: (w1) \Rightarrow b1$

$A_2: (w2) \Rightarrow b2$

$A_3: (w3) \Rightarrow b3$

$A_4: A_2, A_3 \rightarrow \neg b1$

$A_5: A_1, A_3 \rightarrow \neg b2$

$A_6: A_1, A_2 \rightarrow \neg b3$

Why Restricted Rebut

$S = \{ w1; w2; w3; b2, b3 \rightarrow \neg b1; b1, b3 \rightarrow \neg b2; b1, b2 \rightarrow \neg b3 \}$

$D = \{ w1 \Rightarrow b1; w2 \Rightarrow b2; w3 \Rightarrow b3 \}$

$A_1: (w1) \Rightarrow b1$

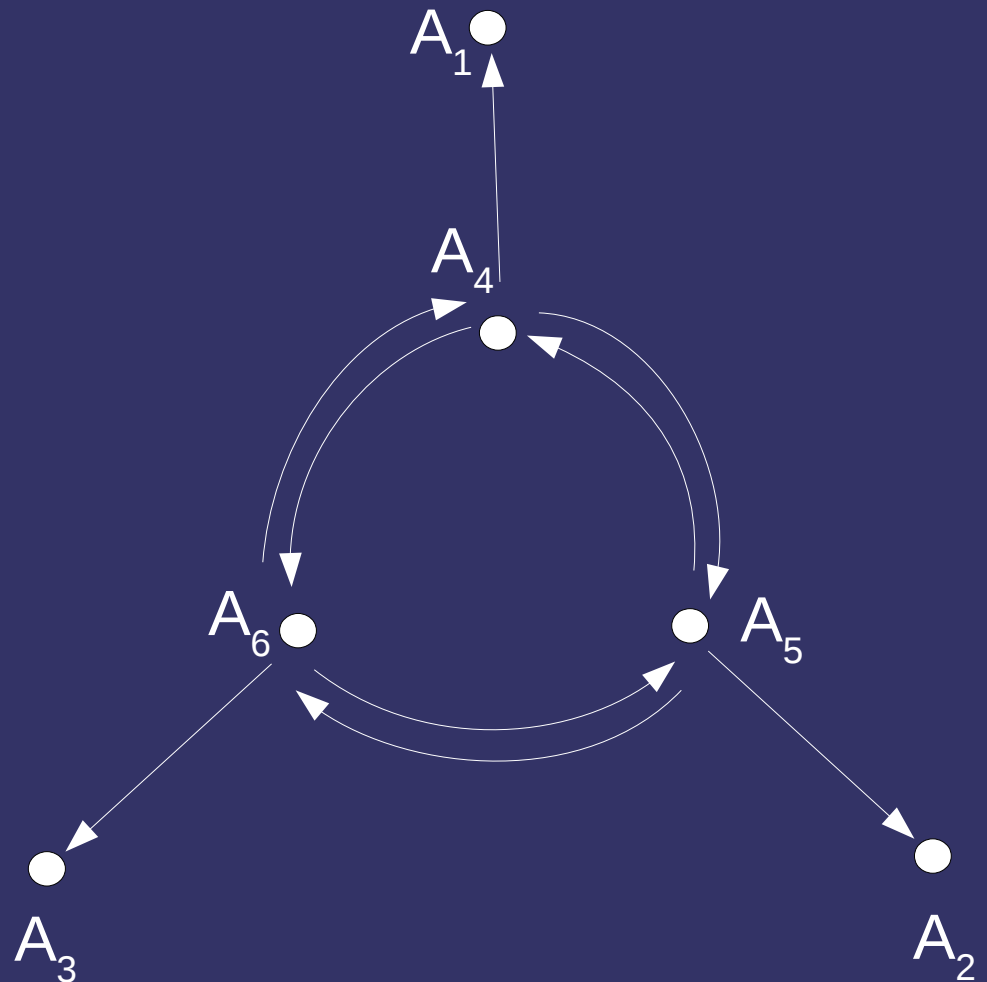
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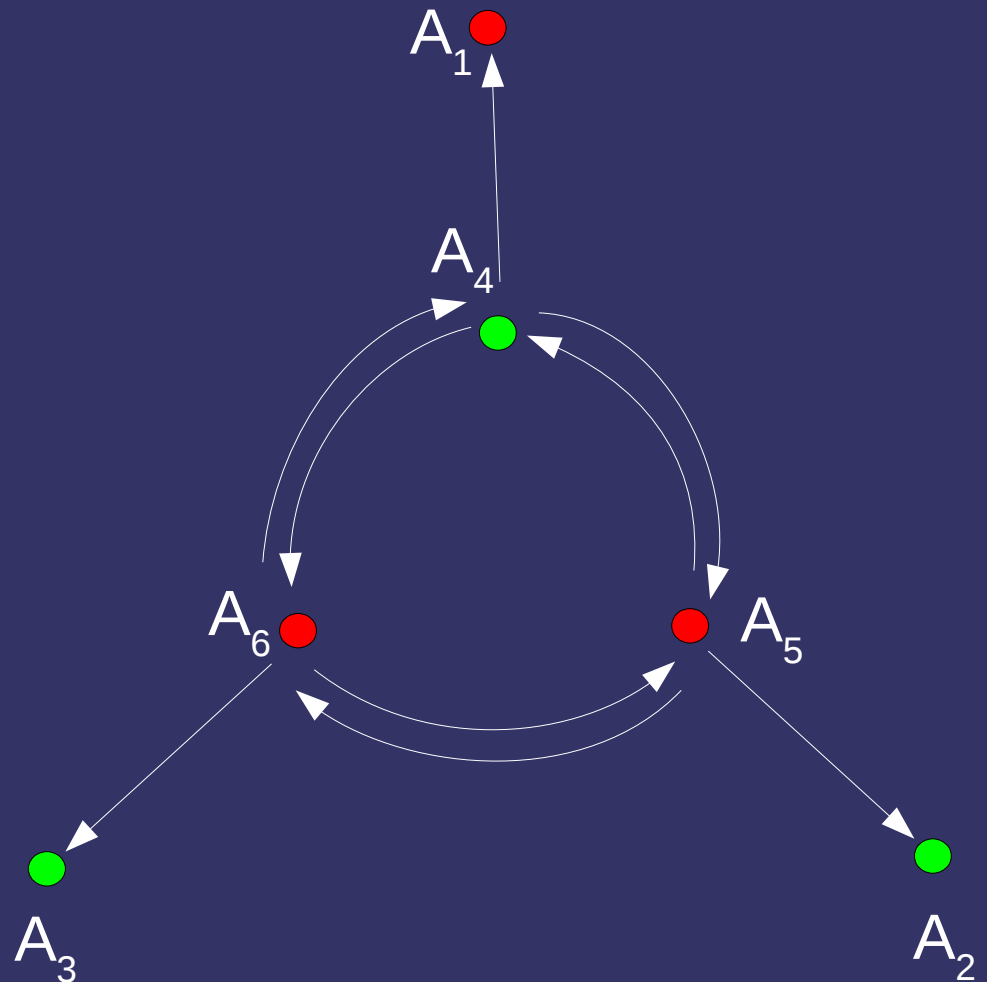
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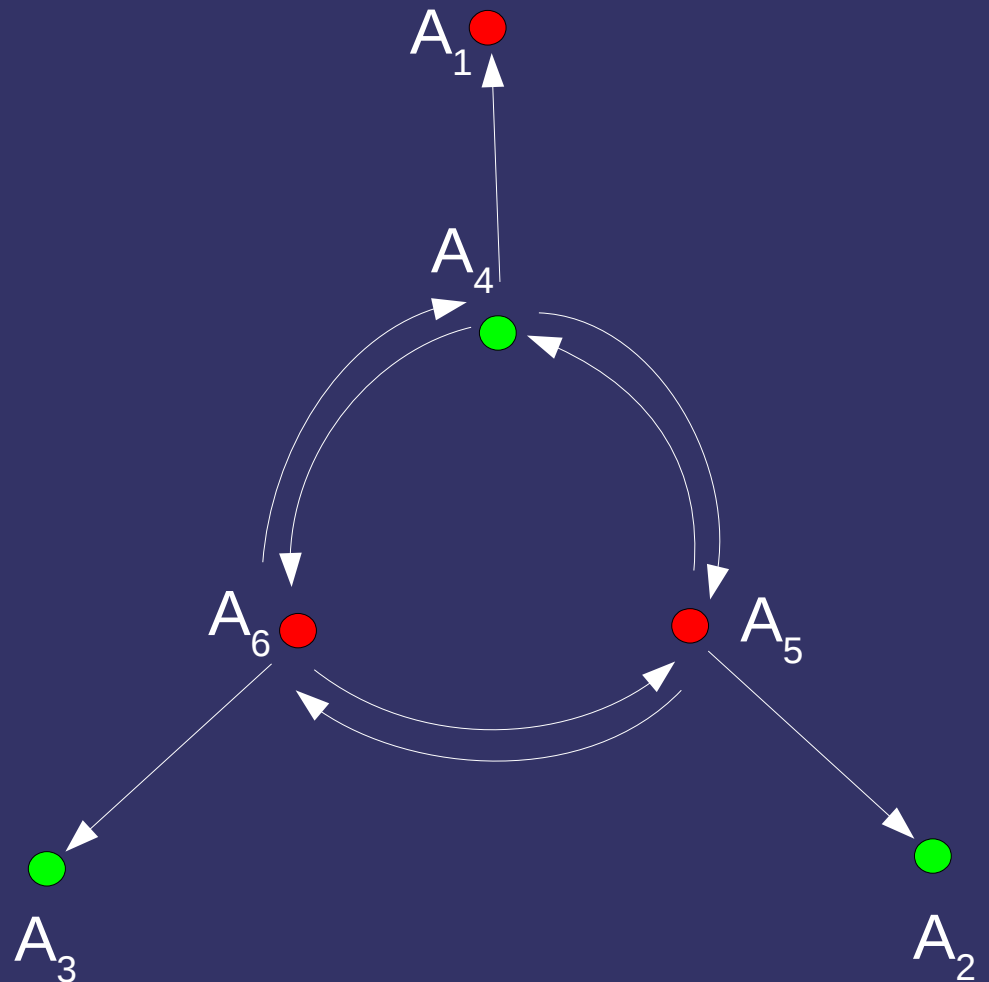
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$b1$ $b2$ $b3$ $\neg b1$ $\neg b2$ $\neg b3$



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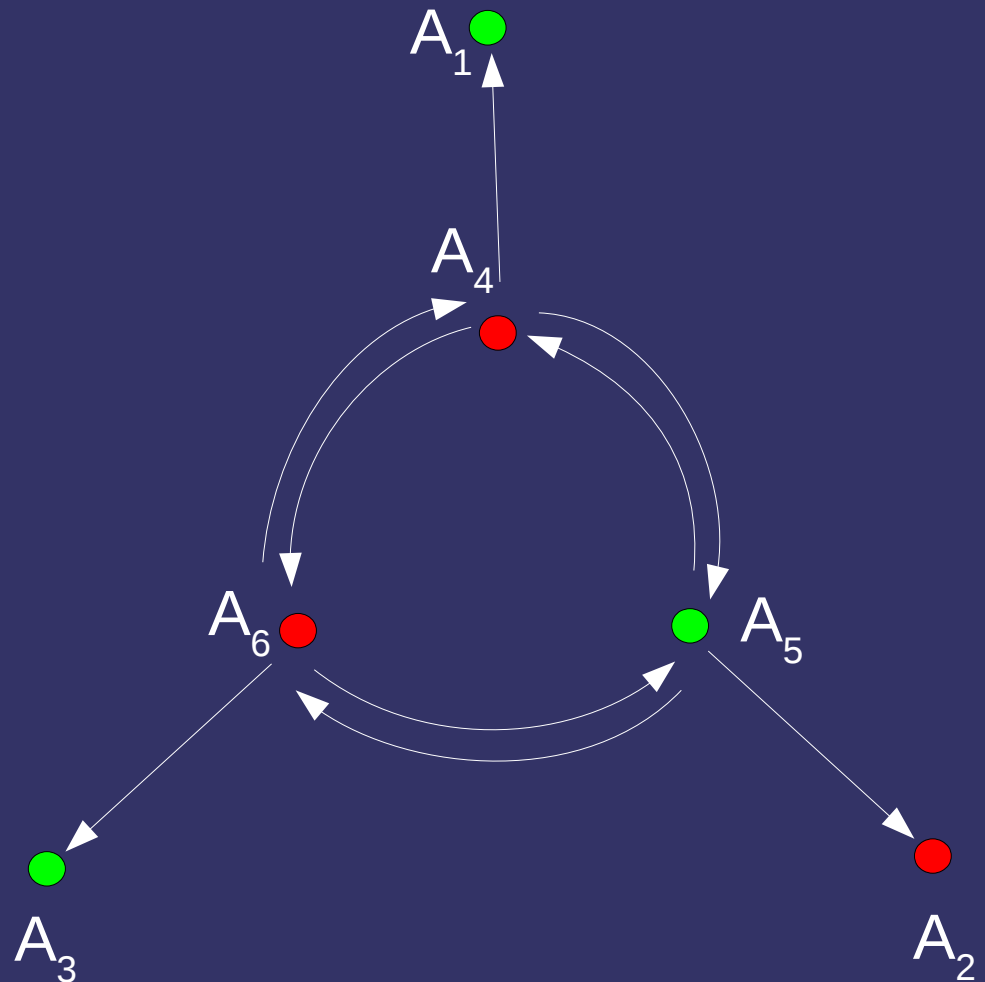
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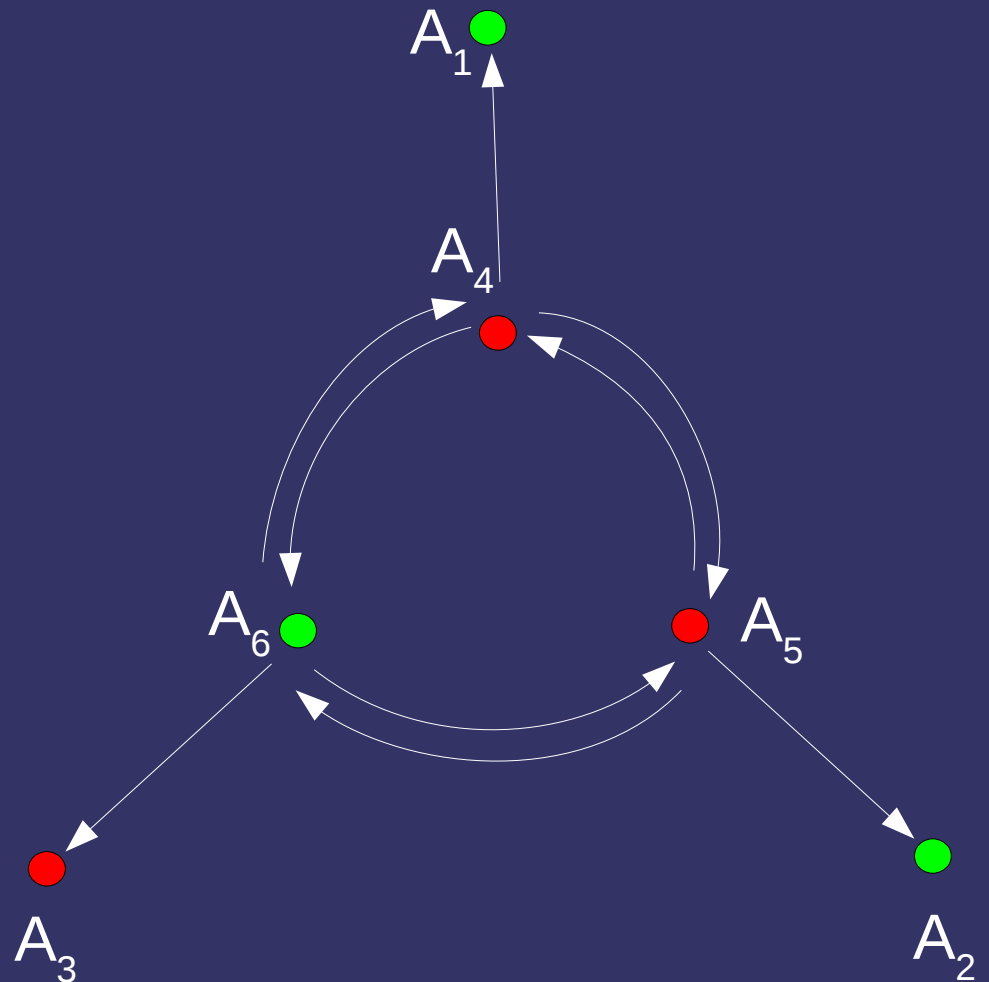
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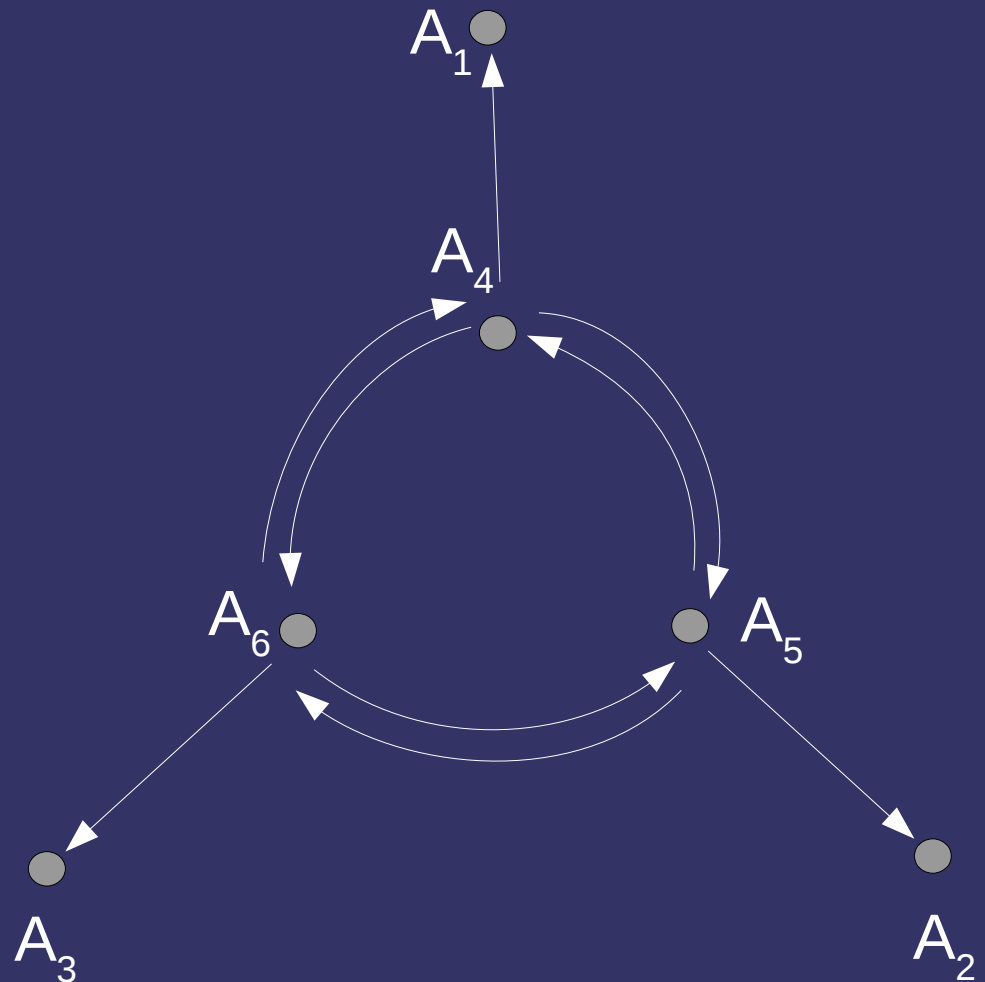
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b1 b2 b3 $\neg b1$ $\neg b2$ $\neg b3$



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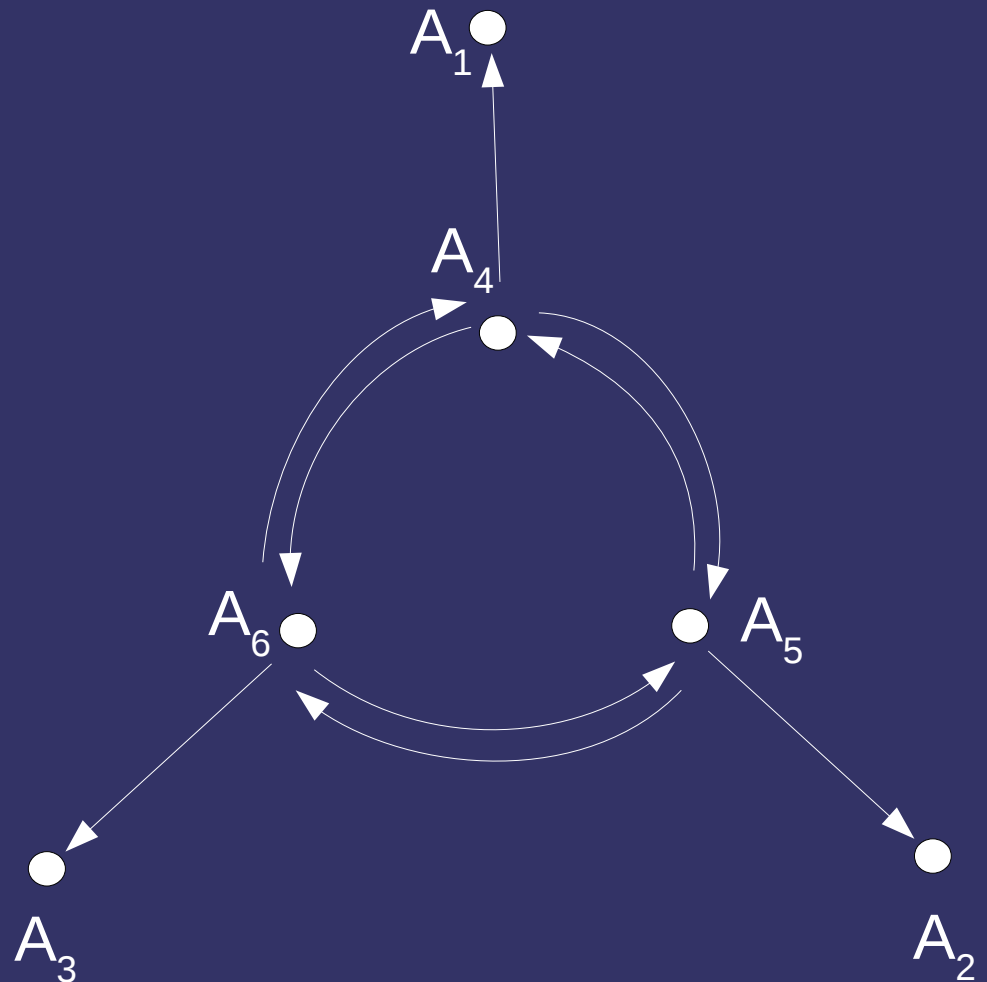
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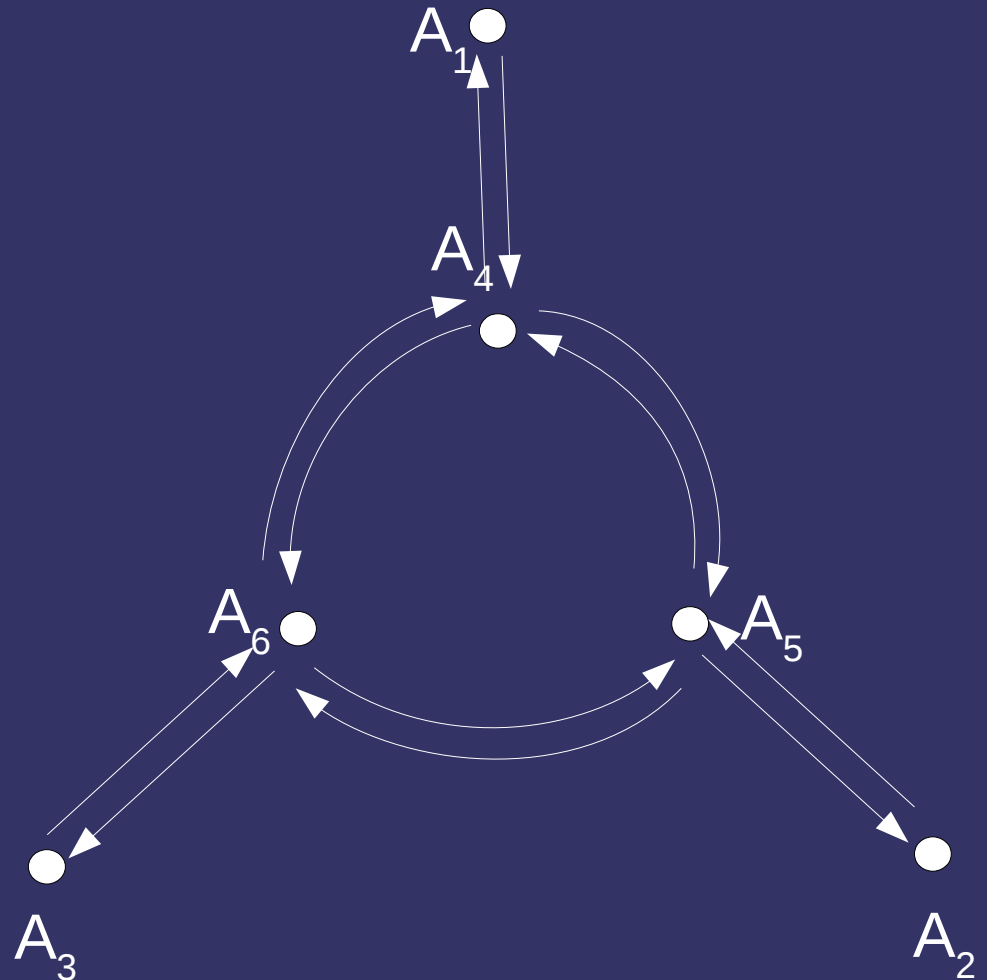
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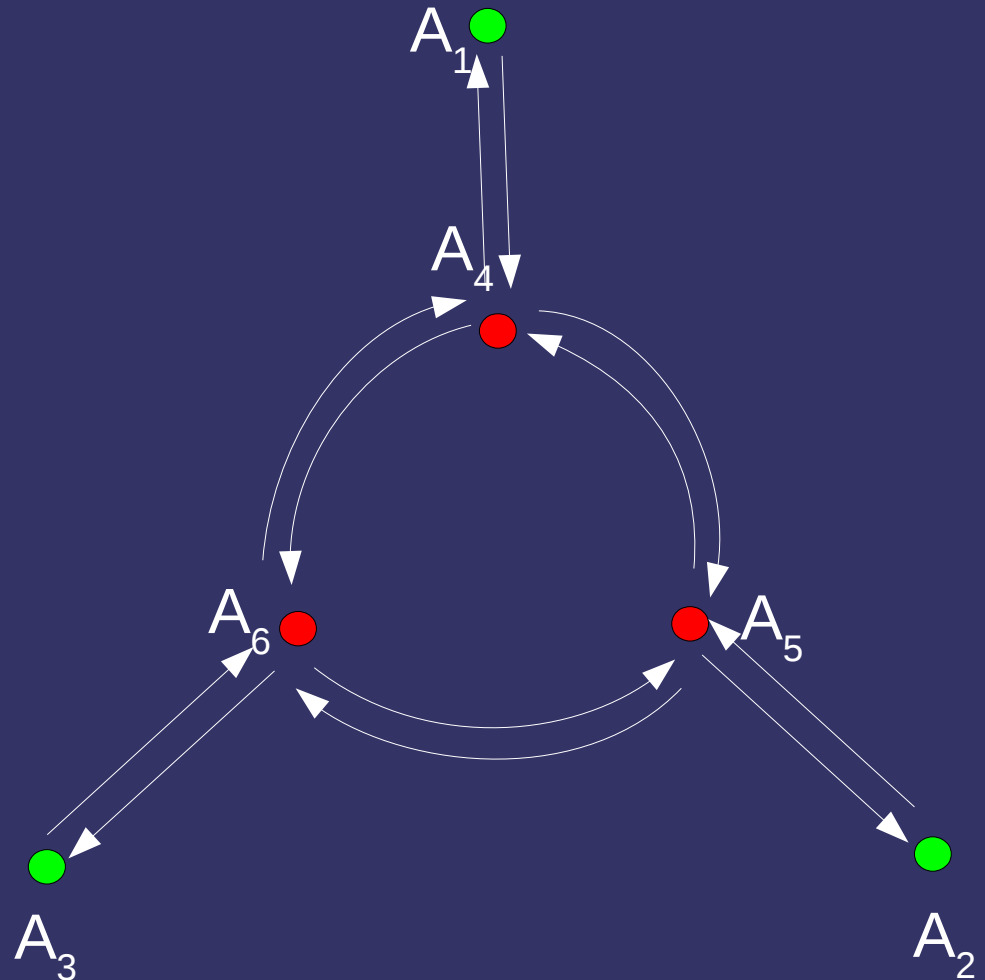
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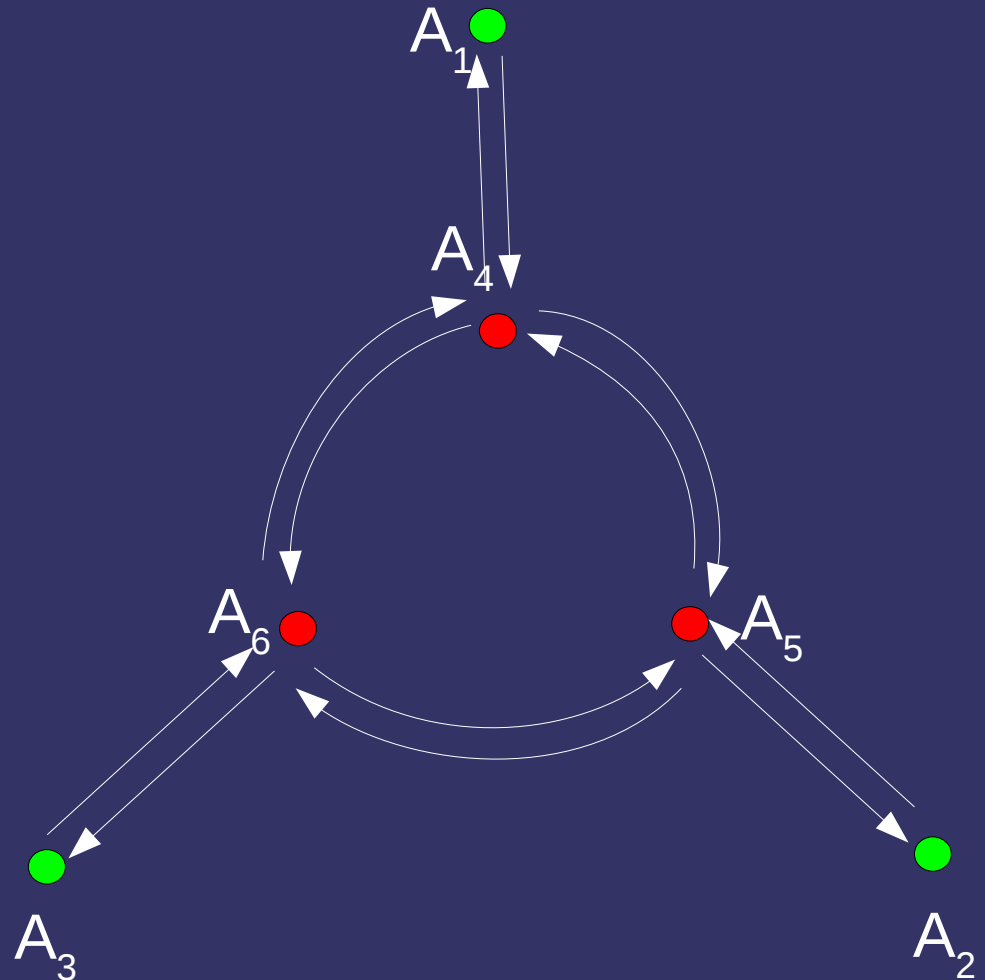
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$b1 \ b2 \ b3 \quad \neg b1 \ \neg b2 \ \neg b3$



Why does this matter?

- Philosophy (as well as other science) aims to find statements that can be defended
- “*can be defended*” is usually left vague
- “*AI makes philosophy honest*” (Daniel Dennett)
- If you try to make things precise, all kinds of problems pop up...

What we found so far...

- when the aim is to apply unrestricted rebut and still have consistent outcome, only the *grounded* labelling will do (not preferred, stable, complete, ideal or CF2 labellings)
- a sound and complete discussion game for grounded labellings is available (details tomorrow)
- rule-strength ordering has to be either:
 - the empty ordering, or
 - a total ordering